MATHEMATICS, REASON & RELIGION

JAVIER LEACH
Universidad Complutense de Madrid

ABSTRACT: This paper will study the relationship between mathematics and religion from the perspective of reason and the role played by reason in human knowledge. Firstly, I will study the relationship between reason, logic and mathematics. From this starting point, I will study the relationship between reason and natural science and finally, I will draw some conclusions on the relationship between reason, philosophy and theology. The relationship between mathematics, reason and religion will be studied within the context of the global unity of human knowledge. This paper intends to explain how the 'pure deductive reason' is present in all human thinking. Mathematics and natural science share this universal presence with metaphysics and religion. Pure deductive reasoning is somehow an absolute value that transcends all aspects and levels of human knowledge, including metaphysical and religious knowledge. Metaphysical and theological arguments need to be able to span different cultural communities. Pure deductive reasoning is a kind of reasoning that can fully span communities and it forms a basis for inter-disciplinary, inter-cultural and inter-religious communication.

KEY WORDS: mathematics, reason, pure deductive reason, logic, natural science, metaphysics, religión, revelation.

Matemática, Razón y Religión

RESUMEN: En este artículo voy a estudiar la interrelación entre matemática y religión desde el punto de vista de la razón y del papel de ésta en el conocimiento humano. En primer lugar estudiaré la relación entre la razón, la lógica y la matemática. Partiendo de ahí estudiaré la relación entre la razón y las ciencias de la naturaleza y por último sacaré conclusiones acerca de la relación entre la razón, la filosofía y la teología. Estudiaré la relación entre matemáticas, razón y religión dentro del contexto de la unidad global del conocimiento humano. Este artículo pretende explicar la presencia y el rol de la 'pura razón deductiva' en el conocimiento humano. La matemática y las ciencias de la naturaleza comparten esa presencia universal con la metafísica y la religión. La pura razón deductiva es de algún modo un valor absoluto que trasciende todos los aspectos y todos los niveles del conocimiento humano, incluyendo el conocimiento metafísico y religioso. Los argumentos metafísicos y religiosos necesitan ser capaces de alcanzar a las distintas comunidades culturales. La pura razón deductiva es capaz de alcanzar a todas las comunidades y constituye una base para la comunicación inter-disciplinar, inter-cultural e inter-religiosa.

PALABRAS CLAVE: matemática, razón, pura razón deductiva, lógica, ciencias de la naturaleza, metafísica, religión, revelación.

Introduction

At first sight, it appears that mathematics and religion are two different and unconnected topics; that the one has nothing to do with the other. History however demonstrates how many thinkers have discovered different kinds of relationship between mathematics and religion. We know that the Pythagoreans gave certain mathematical structures a religious meaning [1]. In the last few years,

[1] Priya Hemenway, *Divine Proportion: Phi In Art, Nature, and Science*, Sterling Publishing Company Inc., 2005, p. 56.

some interesting ideas have surfaced on the relationship between mathematics and religion[2].

I will study the relationship between mathematics and religion from the perspective of reason. The concept of reason has been used in various ways in different contexts. Generally speaking, reason is linked to the human ability to structure, assimilate and convey knowledge. Of all the sciences, it is mathematics and logic that most clearly and precisely use reason. Developments in the last 150 years in both the fields of mathematics and logic have shown that we can consider mathematical logic to be a part of mathematics. Through mathematics and logic, reason has taken on great importance in scientific knowledge. But reason is not only linked to mathematics and logic. As we will see throughout this paper and in broad terms, reason is linked to the ability to structure and make conveyable all kinds of human knowledge.

I will consider the relationship between mathematics, reason and religion within the context of the global unity of human knowledge. Within the unitary and global reality of human knowledge, I will aim to uncover the specific role played by reason. Two poles or extremes can be identified within human knowledge. Using a geometric image, human knowledge can be illustrated as a sphere or an ellipsoid with two hemispheres and two poles. One of the hemispheres of knowledge unifies reason and the other hemisphere contains multiple perceptions through which knowledge receives new experiences and senses. Logic, mathematics and language are inside the hemisphere of reason. In this hemisphere of reason, knowledge is actively organised and can be conveyed. The other hemisphere of human knowledge is the hemisphere of experiences, perceptions, observations, feelings and emotions. In this hemisphere, called the hemisphere of experience or perception, knowledge is receptive and increases with new perceptions. It is impossible to separate these hemispheres without destroying the internal unity of knowledge. Without the hemisphere of reason, human knowledge becomes obscure and cannot be conveyed. Without the hemisphere of experience, knowledge becomes isolated and impoverished.

This paper will essentially study knowledge from the pole of reason, which I will call the pole of pure deductive reason. I will consider the importance and influence of this pole not only in relation to the areas or aspects of knowledge within the hemisphere of reason that are directly under its influence, but also in relation to the areas and aspects within the other hemisphere of knowledge that depends more directly on experiences and perceptions. Finally, I will pay special attention to the relationship between reason and experience in global philosophical and religious intuition and views.

This paper will continuously bear in mind both the global unity of human knowledge and the bipolar approach to this knowledge. It is important to remember however that this is not a simple bipolar approach whereby the specific

[2] Russell W. Howell and W. James Bradley (eds.), *Mathematics in a Postmodern Age: A Christian Perspective,* Wm. Eerdmans Publishing Co., 2001; John Byl, *The Divine Challenge: on Matter, Mind, Math and Meaning*, Banner of Truth Trust, 2004.

characteristics inherent in each aspect of knowledge can be fully explained from just two poles. The general bipolar nature of knowledge can also be seen in the individual and specific areas of knowledge, such as mathematical knowledge or logical knowledge, which are also bipolar. Although logic and mathematics are areas of knowledge within the hemisphere of reason, we will also discover an internal local bipolarity: one pole is more intuitive, receptive and somehow more experiential and the other pole is more active and rational (Figure 1). This local bipolarity of logic and mathematics will help us to clarify the internal nature of the rationality inherent in logic and mathematics. This analysis of logical and mathematical rationality will lead us to the final extreme of the pole of reason, which I will call the pole of pure deductive reason. Pure deductive reason is located at the edge of the hemisphere of reason and its ability to rationally clarify and structure not only effects logic, mathematics and other realities that are within the hemisphere of reason, such as natural languages, but also all human knowledge as a whole and especially philosophical and religious knowledge.

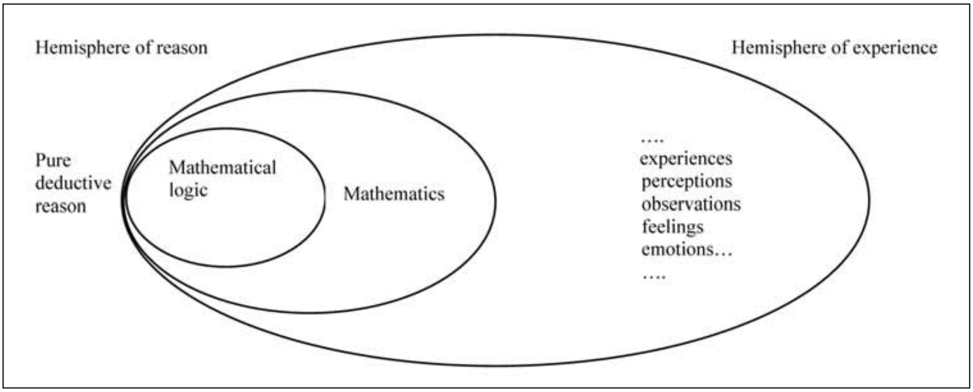

FIGURE 1

The first part of this paper will study the nature of mathematics and mathematical logic as realities that fall within the hemisphere of reason, but which are likewise bipolar. Mathematics also has two poles and two hemispheres (Figure 2): the different kinds of logic responsible for providing internal coherence to the many mathematical intuitions are in the rational mathematical hemisphere. The other, more receptive, hemisphere contains mathematical intuitions, received and taken in by the mathematician in a contemplative or passive way. Mathematical intuitions begin to take on a structure when they become statements and axioms in a mathematical language. There are also two hemispheres and two poles in relation to logic (Figure 3). Pure deductive reason is at one extreme. At the other extreme there are different logic-based intuitions and views that are also expressed in logical statements and axioms, which in turn can be analysed in a standard way from the pole of pure deductive reason. I will show how the pole of pure deductive

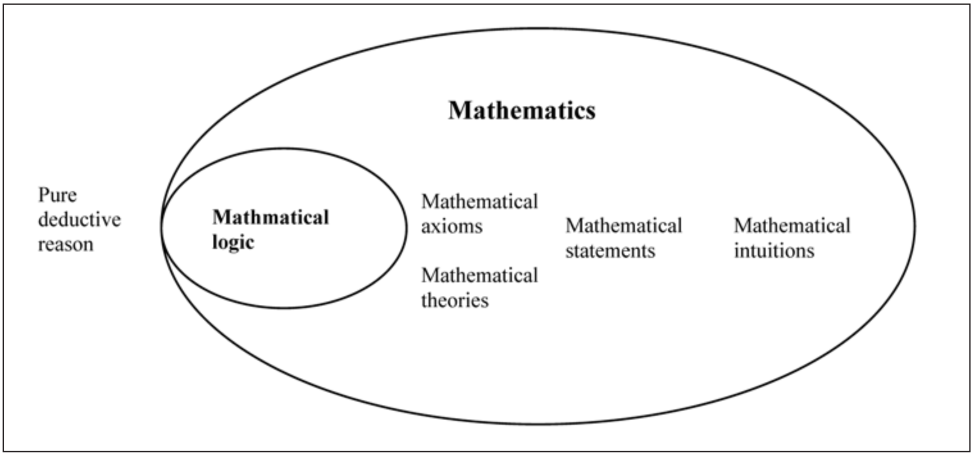

FIGURE 2

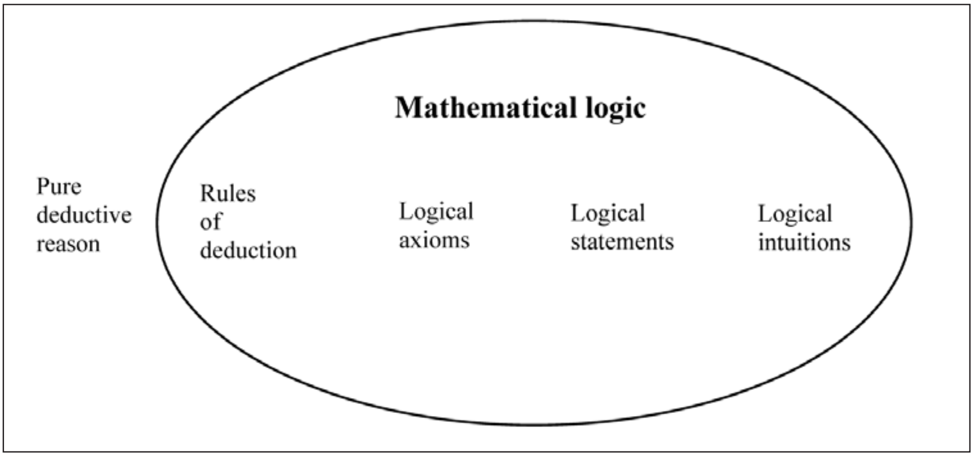

FIGURE 3

reason not only brings dynamic unity, clarity and objectivity to logic and mathematics, but also how it brings dynamic unity, clarity and objectivity to the rest of human knowledge. The dynamic unity and clarity demonstrated by the pole of pure deductive reason cannot be separated from the plurality and complexity inherent in the different kinds of experiences, intuitions, perceptions and feelings required to produce all kinds of knowledge. I intend to show how in mathematics, empirical science and theology, deductive reason interlinks a variety of fields where there are many intuitions, perceptions, observations, views, feelings and emotions. In summary, I will aim to show how the hemisphere of reason is based on a pole or extreme of pure deductive reason, which in turn gives structure to a large amount of human knowledge, both scientific and religious.

The second part of the paper will consider the rationality of empirical knowledge. Empirical knowledge (Figure 4) also has two poles and two hemispheres. The hemisphere of reason contains logic and mathematics. The hemisphere of experience contains scientific observations. In the centre, connecting both hemispheres, are scientific hypotheses, which will form the basis of scientific and mathematical theories once they have been formulated in a suitable language.

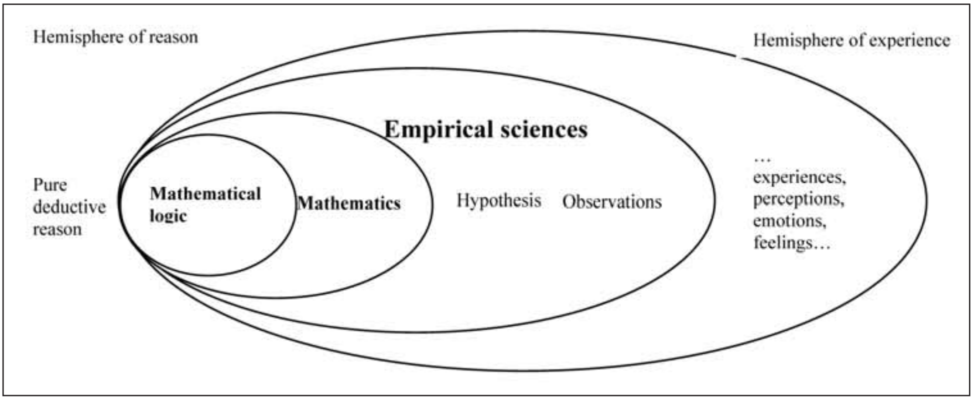

FIGURE 4

In the third part of the paper, I will consider the rationality of global religious views. As a result of their own nature, global religious views need to be conveyed and they therefore need to be reasonable. The quasi-absolute nature of the pole of deductive reason has meant that at times, reason is seen to compete with and antagonise religion, while at other times, reason has helped support different religious views.

1. THE HEMISPHERE OF REASON

Firstly, we need to be aware that when we talk about reason, we are not always saying the same thing; the meaning of the word varies. Reason has been linked to language, to logic, to the mind, to the conscience, etc. Distinctions have also been made between theoretical reason, which considers how things are, and practical reason, whereby we justify our actions. In this paper, I will try to provide a clear, simple and specific view of reason, focusing on mathematics. Before dealing with reason from a mathematical point of view, I will provide a brief summary of some aspects of reason that go beyond mathematics.

Reason and natural language. Mathematics is a formal language developed in close connection with natural languages. Many different natural languages have appeared in the context of different communities and cultures (English, Chinese, Hindu, Arabic, Spanish, etc.). Distinctions can be made between different natural languages as a result of their use of different words and sounds and because they have different grammatical rules that refer to different syntax and semantics. All the different syntax of natural languages however share a relationship with the human race, a common reality among all men and women that enables them to communicate with each other. Reason is an essential characteristic of human communication and languages are the basic tool. Syntactic analysis of languages shows how the rationality shared by the human condition can be seen in syntactic and grammatical similarities between different natural languages. These syntactic similarities demonstrate the unity of human reason. The highest level of unity is shown in the most deeply rooted syntax, where the same structures found in mathematical logic appear in different ways. For example, the conjunctions «y», «pero», «and», «but», «und», and «aber» convey in three different languages and in two different ways the same conjunction used in mathematical logic to connect two statements confirming that the two are true. Another example uses the conjunctions «o», «or» and «oder», which convey in three different languages the same logical disjunction, although here there is a certain ambivalence in the natural languages: for example, in the sentence «Juan prays or smokes», the statement can be considered true both if Juan only prays or only smokes, as well as if he does both at the same time. In the formal languages used in mathematics, ambivalence is not allowed and in the first case an exclusive disjunctive is used and in the second case, an inclusive disjunctive. However, although there is no conjunction in Spanish used in an explicitly disjunctive manner and no conjunction used in an explicitly exclusive way, in Spanish it is easy to explain the disjunction's inclusive and exclusive use. This is the case in all natural languages: although there is no explicit use of a logical mathematical structure in natural language, we can always explain the meaning of logical structures in mathematics in the natural language. The fact that we can explain basic logical structures used in mathematics in any natural language enables us to translate mathematical proofs into any language. To summarise, we can assert that in mathematical logic is present the deeply-rooted rational unity of the different natural languages.

Reason and the human mind. Reason has also been linked to the human mind. The mind is the most specific phenomenon of human nature; in some ways, mental activities could be considered to be the most specifically human of activities. There is a wide-ranging, rich and lively debate around the qualities that characterise the human mind; beyond this debate, we can assert that rationality is a characteristic specific to the human mind. At the same time as they are made up of feelings and emotions, activities of the human mind are governed by basic rational structures that make them coherent and conveyable. The human mind is able to construct rational models that represent reality and to use these models to act in a reasonable way with other human beings, thus changing reality. In order to construct these rational models, the human mind uses concepts to convey perceptions and knowledge; the mind formulates qualities in relation to the things it has perceived; and the mind structures its ideas in a rational way linking concepts. As we will see, there are different kinds of rational model, but mathematics plays a fundamental role in all these models when demonstrating their rational nature.

Rational coherence and the human conscience. Human beings are also characterised by having reached a high level of awareness. A human being's conscience manages to create a whole sense of the world and of itself that distinguishes the human condition from other living beings with a lower level of awareness. The ability to reason is an essential characteristic that allows the human conscience to reach a higher level of internal coherence in the way it perceives the world. The ability to use thought and reasoning to capture the internal coherence of what is perceived is a characteristic specific to the human conscience. The critical rigor inherent in mathematics also demonstrates the highest level of coherence that can be expressed in a rational way.

Theoretical reason and practical reason. Finally, we can draw a distinction between two basic types of human reasoning; reasoning about how things are or about how we should act. Theoretical reason is used to justify our certainty and doubt about how things are. Practical reason is used when we justify our actions.

Theoretical reasoning is linked to statements about how things are. These statements are often called declaratives. For example, «today it is raining» is a declarative statement that attributes the quality of rain to today; «John is clever» is another declarative statement that asserts a quality belonging to John. Using theoretical reasoning, we can deduce declarative statements from other declarative statements. Theoretical reason analyses facts in an impersonal and public way, facts which are in theory accessible to anyone. Natural and social sciences use theoretical reason. Theoretical reason makes practical reason possible. Before we decide to act in the world, we need to know how the world is. Theoretical reason justifies our beliefs about the world, transcendence and God. Practical reason justifies our actions in accordance with our beliefs.

Using practical reasoning we think about how we should act. Practical reasoning is linked to normative statements. For example, «you must eat to live» is a normative statement. Practical reason is used when we justify our actions. Using practical reasoning we justify the options we take in situations where several different options are open to us. Practical reason is about clarifying which option is best. While theoretical reason aims to explain how things are, practical reason aims to assess events adequately in order to determine which option is best. Practical reason assesses and weighs up the facts from an individual and group point of view depending on whether individual or group decisions need to be made.

There are links between theoretical and practical reason and there can be no contradiction between the two. Both theoretical and practical reasoning can be formalised[3] and as a consequence, the critical rigor inherent in mathematics can be applied to both. Although theoretical reason alone is not sufficient to determine our actions and practical reason is also required, for methodological reasons I will focus on theoretical reason in this paper, based on the use of theoretical reason in mathematics.

2. Mathematics and theoretical reason

The study of how mathematics uses theoretical reason will reveal to us the deductive ability of reason, shared by both theoretical and practical reason. The critical rigor inherent in mathematical reason will demonstrate fundamental characteristics of the pole of pure deductive reason, which is located in one extreme of mathematics and logic.

Logic, mathematics and language. We have considered the study of reason as a study of one pole of human knowledge seen as a global unity. Let us focus therefore on the hemisphere containing this pole of reason: this hemisphere contains logic, mathematics and language. More specifically, let's firstly focus on mathematics as the part of language that aims to logically describe and structure mathematical intuitions. As we have said, we can see that there are also two poles and two hemispheres in mathematics. In one mathematical hemisphere there is logic and in the other, mathematical intuitions. In the middle, between the two poles, there are mathematical axioms and statements (Figure 2). Not all mathematics is pure logic. Logicism[4] dates back to the end of the 19th century and the beginning of the 20th century and aimed to reduce mathematics to logic. Logicism failed however and it was proved that mathematical knowledge cannot be reduced to logic. Opposite the pole of logic in mathematics, there is a rich and complex world of mathematical intuitions that cannot be reduced to logic. Unlike empirical intuitions, mathematical intuitions are characterised by their simplicity and clarity and this means that they can easily be structured in a logical way. In fact, it is in relation to mathematical intuitions and within the context of the study of mathematical logic that we find concepts and arguments that can clarify our ideas about the rational extreme of logic, which we call pure deductive reason.

Deductive reason and logical intuition. If we look at Figure 2, we see that mathematical logic is a part of mathematics within the mathematical ellipse in the hemisphere of reason. But there is also an internal tension between two poles in mathematical logic. Mathematical logic itself is not pure deductive reason. In one pole of logic there is deductive reason and in the other logical intuitions

[3] Georg Henrik von Wright, «Norms, Truth and Logic», in Georg Henrik von Wright, *Philosophical Papers I. Practical Reason*, Blackwell, Oxford, 1983; trans. by Carlos Cabrera Alarcón, *Normas, verdad y lógica*, Fontamara, México, 1997.

[4] Gottlob Frege (1848-1925) & Bertrand Russell are the best known advocates of this logistical movement.

(Figure 3). If mathematical logic were pure deductive reason, all logic would be the same or certain logical questions could be narrowed down into others. But not all logic is the same, not all logic accepts the same basic intuitions. The principle of excluded middle for example is accepted by classical mathematical logic, but not by constructivist logic [5]. There is no reason why all logicians have to accept the excluded middle principle [6]. The excluded middle principle is based on an intuition that is accepted by classical mathematics, which constitutes the majority of mathematical statements and proofs, but which is not accepted by constructivist mathematics. The excluded middle principle is a logical axiom and is located between the pole of deductive reason and the pole of logical intuitions (Figure 3).

Pure deductive reason. Pure deductive reason answers questions about the reason for things: *Why?* Pure deductive reason does not assign this question a specific object. The pure question about reason can be applied to any object of knowledge. At the beginning of this paper, I considered the idea of knowledge as a reality containing two extremes or poles and I have also said that this is not a simple bipolar approach whereby everything can be explained using just these two poles. By studying the hemisphere of reason, where we have placed logic, mathematics and language, we have described a new polarities within logic and mathematics that help us to clarify the concept of reason. The pole of pure deductive reason with the question *Why?* is at one edge of the hemisphere of reason and is applied to much logic and mathematics. But pure deductive reason is not only applied to logic and mathematics; pure deductive reason is a pole of all rational knowledge and by studying it we can also clarify other realities of human knowledge and in particular, scientific, philosophical and religious views.

3. The historical development of mathematics

Up until this point, we have broadly discussed reason, mathematics and logic, but without specifying in detail what we are referring to when we speak of mathematics and logic. Below is a more detailed summary of what is meant by mathematics. To do this, we need to look at the historical development of logic and mathematics starting with important historical events and moments that have caused cultural changes in the world of mathematics and which have

[5] The principle of excluded middle asserts that given a statement A, A is either false or true. Constructivist logic does not accept that one of the two possibilities A or not A is inevitably true, only that the two possibilities cannot be true at the same time, as we would be faced with a contradiction. Constructivist logicians can only assert A when A can be proved and can only assert not A when not A can be proved, but if neither A nor not A can be proved, we cannot assert: «A or not A».

[6] In order to understand these ideas it is important to distinguish between the excluded middle principle that states that «A or not A» is true and the principle of no-contradiction that states that «A and not A» cannot be both true.

resulted in the vision we have today of logic and mathematics. This historical summary will help us to see how mathematics has changed and to discover the role that mathematics has taken on over time in human knowledge and also how mathematics has helped to clarify the role of reason in knowledge and, more particularly, the role of pure deductive reason in knowledge. The relationship between mathematics and empirical science has been a very important factor in the historical development of mathematics. As well as its intrinsic value, mathematics has become the basic tool used by natural sciences to express the laws of nature. Defining scientific laws using mathematical statements has provided them and scientific reasoning based on them with the highest level of rigor and accuracy. But the aim of this paper was not to limit myself to just mathematics and natural sciences. Further on, I will comment on the use of pure deductive reason in metaphysics and theology using the way in which reason is understood in mathematics. In this paper, I intend to show that the rigor of mathematical reasoning is not only useful and necessary in natural sciences, but also in metaphysics and theology.

Mathematics in different cultural and historical contexts. In line with both the holistic and bipolar approach to knowledge that I considered at the beginning of this paper, I will not describe mathematics as an isolated science to be studied without relation to other subjects. From a holistic point of view of knowledge, mathematics is not understood as an isolated subject. By studying the relationship between mathematics and other kinds of knowledge throughout the different historical and cultural moments experienced by mathematics, we will have a better understanding of the true nature of mathematics. From a bipolar diversity approach to knowledge, we are however able to isolate mathematics as part of the hemisphere of reason and study it separately. During the first half of the 20th century, much thought was given to mathematics itself. The study of pure mathematics, as considered during the first half of the 20th century, became so independent that it tried to back up mathematics with mathematical methods. The study of mathematics using mathematical methods is called metamathematics. Metamathematical reflection has clarified very important issues about the nature of mathematics. Before mathematics had developed sufficiently and was mature and independent enough for metamathematics to be considered, there was an historic process of growth and development that lasted centuries and which shone much light on the nature of mathematics. Mathematics has grown and developed over different periods throughout history. Historical analysis of the complex development of mathematics helps us to gain a better understanding of what mathematics is in itself and in relation to other knowledge.

Three historical periods of mathematics. I will highlight three historical periods in the development of mathematics: *1. Pre-modern mathematics.* Pre-modern mathematics spanned a historical period that started when man learned to count and measure for the first time up until the advent of modern science around the beginning of the 17th century. *2. Modern mathematics.* Science in the modern age began a new stage in relation to mathematics. The mathematical formulation of the empirical laws of nature is characteristic of modern science. *3. Post-modern mathematics*. This third period is characterised by the formal rigor of mathematics as an independent science capable of being founded in itself (in part).

3.1. *The pre-modern period*

The pre-modern period stands out for two reasons: (a) During this period, mathematics appears as a formal science. Mathematical formalisms are used for the first time. (b) Although the origin of mathematical formalisms cannot be separated from the use of these formalisms in trade, the measuring of fields and in astronomy, the pre-modern period in mathematics is characterised by the fact that mathematics was still not applied to physics as a structured science.

(a) The use of formalisms in the pre-modern period

Thousands of years ago, human beings carried out formal mathematical reasoning. The first historical evidence of signs, lines, knots and other symbols used to represent numbers dates back around 50,000 years[7]. This evidence proves the existence of primitive mathematical reasoning. Similar formal reasoning has been applied for thousands of years in different situations. Man began to use mathematical reasoning when he started to represent the first numbers 1, 2, 3, etc., with different kinds of formalisms and to convey these formalisms with words. When numbers were first expressed vocally, different words were probably used to represent different objects. The word «two» was not the same when used to mean two men or two horses for example. In English, we still distinguish between: *Team* of horses, *span* of mules, *yoke* of oxen, *brace* of partridge, *pair* of shoes, *couple* of days[8]. Words became standardised and abstract calculations began to made, such as three minus one equals two, which could be used in different situations: if I have three apples and I give two to somebody, then I will be left with one; the same will happen if I have three fish and I give two to somebody.

Formal axiomatic methodologies appear in the pre-modern period of mathematics. Euclid's Elements (323 BC to 283 BC) is a geometric treatise that introduces geometric statements that can be deduced from a small set of axioms. Geometry that is deduced from the axioms in the Elements is currently called Euclidean geometry. The deductive axiomatic method presented by Euclid in the Elements is still the most common method in mathematics of deducing statements using axioms. The deductive axiomatic method begins by defining a set of axioms whose truth is obvious and using these axioms, other statements

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[7] HOWARD EVES, *An Introduction to the History of Mathematics,* Ed. Saunders College Publishing, 1992, p. 9.

[8] HOWARD EVES, *An Introduction to the History of Mathematics,* Ed. Saunders College Publishing, 1992, p. 11.

in a theory can be deduced. Euclidean geometry forms part of the foundations of current geometry studies. In the 19th century, Nikolai Ivanovich Lobachevski (1792-1856) developed non-Euclidean based geometry for the first time. Lobachevski's geometry was different from Euclid's in that it accepted the first four axioms in Euclid's geometry, but rejected the fifth. Nowadays, Euclidean and non-Euclidean geometry co-exist as two kinds of geometry that are deduced using two different sets of axioms.

Mathematical axioms. Mathematical axioms are formal postulates that can be accepted or rejected, but they cannot be falsified by an observation. Mathematical axioms are accepted as true because they are perceived as such by mathematical intuition, but there is no argument, regardless of intuition, that requires an axiom to be accepted or rejected. For example, the appearance of non-Euclidean geometry does not negate the validity of Euclidean geometry. There is no empirical observation that confirms one kind of geometry and falsifies the other. Mathematical axioms are independent of empirical observation. The fundamental difference between mathematical axioms and scientific hypotheses inherent in empirical science, is that mathematical axioms are accepted on the basis of mathematical intuitions, whereas scientific hypotheses are induced from experience and are confirmed by other new experiences and by their logical coherence to other scientific hypotheses. As we will see however, although mathematical axioms belong to a world that is autonomous and independent of empirical observation, the mathematical theories deduced from them can be applied to empirical explanations. Whereas traditional Euclidean geometry can explain the empirical phenomena of Newtonian physics, Einstein's theory of relativity is explained using non-Euclidean geometry.

Formal logic. In the pre-modern period, a formal methodology began to be developed for logical arguments. Aristotle was the first philosopher to study the formal laws of logic in a systematic way. In fact, the laws of logic suggested by Aristotle, together with later contributions by stoic philosophers, are almost the only systematic and scientific study of the laws of logical reasoning up until the 19th century.

Formal logic and language syntax. Aristotle suggested a certain kind of formal argument called syllogism. A syllogism contains two statements called premises, which have a certain syntactic shape to them and a statement called a conclusion, which also has a certain syntactic shape. The conclusion is only deduced from the premises because of the syntactic shape of the premises and the conclusion. Aristotle's syllogisms and the stoic philosophers' formal logic were based on the syntactical shape of language. The development of formal logic after Aristotle and the stoic philosophers was almost non-existent until the beginning of the post-modern period of mathematics towards the middle of the 19th century.

Categorical logic. Aristotle's logic is categorical in the sense that for Aristotle, a statement about a 'fact' is either true or false. Although we may not know whether the statement is true of false when we make it, the statement is always 'objectively' true or false. In other words, it is true or false regardless of whether we know it or not. In Metaphysics[9] Aristotle defines truth as: «To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, and of what is not that it is not, is true». According to this classical view of the truth, a statement is true when we can form a correspondence between what is asserted and the 'object' about which we are asserting something. This classical view of the truth is also called truth as correspondence, because it is based on the correspondence between a statement and the object to which the statement refers.

Truth as correspondence. The view of truth as correspondence characterised mathematics up until the 20th century. Logical positivism took on this view particularly in the first half of the 20th century. As we will see, this view of the truth caused problems during the post-modern period in mathematics. The problem lies in the fact that the definition of truth as a correspondence between a statement and an object assumes that reality is made up of a set of objects about which we can make a set of statements that will be true or not, according to whether the statements correspond to the objects referred to or not. In other words, this view of the truth as correspondence assumes that reality is made up of a set of different and distinguishable objects.

And the assumption that reality is made up of a set of different and distinguishable objects caused problems in the post-modern period for both mathematical and empirical reasons.

(b) The division of mathematics and physics in the pre-modern period

Mathematics and physics. During the pre-modern period, the relationship between mathematics and physics was still weak. Aristotle's physics (384 BC to 322 BC) was very important in the pre-modern period. In fact, Aristotle's physics was taught in Europe until the 17th century. Aristotle's physics does not study the causes of physical phenomena using quantitative methods and mathematics, as happens in modern physics. Based on a modern view of speculative, non-empirical criteria, Aristotle distinguishes between four causes: material, formal, efficient and final. The empirical and mathematical study of modern physics has brought together these causes. Aristotle's' works were studied and spread by Arabic philosophers like Avicena (980-1037), Avempace (1080-1138) and Averroes (1126-1198). Thomas Aquinas (1225-1274) incorporated Aristotle's philosophical beliefs into Christian theology. For purely speculative reasons, Aristotle thought that the laws governing the movement of stars were different from the laws governing the movement of earthly bodies.

Mathematics and reality. The first physicist to discredit Aristotle was Galileo (1564-1642), based on modern scientific criteria. Empirical observations and

[9] ARISTOTLE, *Metaphysics*, 1011b25.

mathematical calculations led Galileo to assert that the earth was just another planet. Galileo defended a mathematical explanation of the laws of nature: *Philosophy is written in this grand book, the universe, which stands continuously open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles and other geometrical figures, without which it is humanly impossible to understand a single word of it; without these one is wandering about in a dark labyrinth* [10].

3.2. *The modern period*

The modern period was characterised by the use of mathematics to describe the scientific laws of nature. Mathematics was first used to describe the laws of physics and later the quantitative mathematical methods that have crossed over into other sciences such as chemistry, biology and sociology, etc.

Empirical hypotheses and mathematical axioms. Isaac Newton (1643-1727) demonstrated that the mathematical laws governing mechanics are the same on earth as those that govern celestial bodies. Newton's law of universal gravitation can be applied to all bodies. The uniformity of this law is a scientific hypothesis. Unlike Euclid's mathematical axioms, the law of universal gravitation can be falsified by subsequent observations and it was indeed falsified, as understood by Newton, by observations explained using the laws of relativistic mechanics.

Rational optimism. Mathematics is a formal science and the laws expressed through mathematics are mechanic and automatic. Alongside Newton, Gottfried Wilhelm von Leibniz (1646-1716) developed infinitesimal calculus with a possibly clearer formal notation, which helped its development. Leibniz' huge formal and mathematical ability meant that he took modern optimism as far as possible, being able to explain mathematically all of nature's laws. Leibniz' optimism led him to express the principle of *sufficient reason,* which states that all events can be explained. The principle of sufficient reason understood in mathematical language represents the optimistic side of the mathematical representation of the laws of nature. Formal optimism also led Leibniz to think that all debates between two willing people could be resolved by formalizing the arguments. Leibniz' optimism led to a logical belief in the causal determinism of the laws of nature. The narrowing down of causes to mathematics and mathematical optimism led to an almost inevitable belief in causal determinism.

Causal determinism. Pierre-Simon Laplace [11] (1749-1827) believed in causal determinism. Laplace believed that, just as astronomical phenomena could be

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[10] GALILEO GALILEI, *Opere,* 4, 171 (translation into English as quoted by Machamer in the Cambridge Companion to Galileo, p. 64f).

[11] Laplace developed Newton's mechanics. Laplace's equations are important, as is his partial differential equation.

predicted by Newton's laws, all phenomena could be predicted using the location and momentum of the atoms making up material. Laplace explained his determinist view by saying that if there were a demon that new the location and momentum of all the atoms in the universe at any given moment, this demon would be able to predict all future events using Newton's equations.

Wide variety of scientific disciplines. Throughout the modern period, mathematics was applied to other sciences such as chemistry, biology, geology and many branches of medicine. Units of measurement for volume, length, time, intensity of electric current, temperature, light, etc. were established and mathematics was incorporated into more and more areas of scientific knowledge.

3.3. *The post-modern period*

The most characteristic feature of mathematics in the post-modern period is possibly the development of the deductive reason of logic.

The deductive reason of logic. Formal logic had not developed further since Aristotle. The causal determinism of the modern period assumed that the causes described by mathematical laws created a complete system of causes through which each of its effects could be predicted. But this assumption was not proved. Modern mathematicians did not understand the logical rules governing mathematical language and the semantics of mathematics was not clear either. Neither the semantics nor the syntax of the mathematical language was accurately known.

Two levels of language analysis. A first level of analysis of classical mathematical logic is the logic of propositions. A second and more subtle level is the logic of predicates.

The logic of propositions. The logic of propositions was studied by the stoic philosophers. George Boole (1815-1864) carried out the first full study on the logic of propositions using formal algebraic methods [12]. Propositional logic only analyses mathematical statements up to the level of basic atomic propositions. An atomic proposition is a mathematical statement that can be either true or false and is not made up of other more simple atomic statements. Atomic propositions are connected using syntactic connectives, thus forming molecular propositions. «And», «or», «only when» are examples of syntactic connectives. For example, «John eats and Louise dances» and «John eats only when Louise dances» are two molecular propositions formed using the basic atomic propositions «John eats» and «Louise dances». From an atomic physics point of view, the logic of propositions

[12] GEORGE BOOLE, *The mathematical analysis of logic,* New York: Philosophical Library, 1948 (first published in 1847. Cambridge: Macmillan, Barclay, & Macmillan; London: George Bell); GEORGE BOOLE, *An Investigation of the Laws of Thought,* Prometeus Books, New York, 2000 (first published in 1854).

only analyses language up to the level of atoms, without analysing electrons and elementary particles.

The logic of predicates. The logic of predicates analyses the inside of atomic statements. Inside atomic statements, the logic of predicates discovers quantifiers that refer to a certain domain and predicates whose meaning is also interpreted in a certain domain. The quantifiers required to express all mathematical statements are the universal «all» (∀) and the existential «exists» (∃). Aristotle had already used universal and existential quantifiers as the basic elements of syllogisms. In 1879, Gottlob Frege (1848-1925) published *Begriffsschrift*[13] (Concept Script) with the subtitle: «a formal language of pure thought modelled upon that of arithmetic». *Begriffsschrift* contains the first formal logical system to take on board all deductive reasoning in mathematics. He introduced the quantifiers ∀, ∃ and specific symbols for logical relationships. Concept Script enabled logical inferences to be represented as formal mechanical operations based only on the symbols themselves.

Semantics of mathematical language. The logic of predicates as formulated by Frege offered syntactic foundations for mathematics. But these foundations were still not specific, unlike the models referred to in mathematical language. The objects referred to in mathematical statements that make the statement either true or false were not specified. Georg Cantor (1845-1918) uniformly described all mathematical objects as sets. A set is defined as a group of its elements, which at the same time can be other sets or original elements. Cantor defined a set as «a collection into a whole of definite and separate objects of our intuition or our thought». The idea of collection helped Cantor to express what mathematical objects are in a uniform way: collections of elements. As a result of set theory, mathematical logic acquired standard semantics. Mathematical logic could deal with numbers, points on a map and the seconds in the day in the same way as sets. All objects that can be dealt with by mathematics are collections of objects and mathematics deals with them as collections of objects.

Paradoxes. The formal explanation of the entity of mathematical objects using set theory came up against Russell's paradox. According to Cantor's definition, if P is a well defined property, we can form a set of all elements with the property P. Russell's paradox is as follows: Q is a set containing all sets that are not elements of themselves. Q is a set according to Cantor's definition, as it collects together in a whole objects that contain a property P. If Q is a set, we will know whether Q is an element of Q or not. If however Q is an element of Q, we have to conclude that Q is not an element of Q and if Q is not an element of Q, we have to conclude that Q is an element of Q. To avoid Russell's paradox, restrictions were placed on the cases when a well defined property defined a set. A well defined property can only define a subset of a set. With this restricted definition, all mathematical objects can be constructed avoiding Russell's paradox.

[13] GOTTLOB FREGE, *Begriffsschrift: eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*, Halle, 1879.

The search for completeness. Avoiding mathematical paradoxes prevented contradictions, but this did not go far enough to test the complete causal determinism suggested by modern science. In 1920, David Hilbert (1862-1943) aimed to prove that all mathematical statements can be deduced using a set of axioms in a consistent way, in other words, without arriving at a contradiction. For Hilbert, mathematical axioms were statements about signs whose meaning is clear prior to all argument [14]. 0 and 1 for example are signs and when an element belongs to a set it is also a sign because it's meaning is clear prior to all arguments. An example of an axiom in set theory is the statement that says that if two sets contain the same elements, then they are the same. By proving that all mathematics could be consistently deduced using a set of axioms, which at the same time could be mechanically built, Hilbert aimed to provide mathematics with complete deductive reason. If Hilbert's programme was successful, all mathematics could be deduced with complete formal rigor using certain obvious axioms for mathematical intuition.

Incomplete deductive reason. Kurt Gödel (1906-1978) proved that whenever arithmetic axioms do not lead to a contradiction (i.e. a proposition of the $A \wedge \neg A$ kind), there will be a proposition U, which is valid in arithmetic models but which cannot be deduced in the arithmetic system. In other words, it cannot be effectively deduced in the formal arithmetic system if U is deduced from axioms or not. Therefore, we cannot «decide» if U belongs to the system or not. Gödel also proved that if arithmetic is consistent, the formal statement that expresses arithmetic consistency can not be proved within this arithmetic system. Gödel's incompleteness theorem proves that deductive reason applied to arithmetic proves the incompleteness of any axiomatic arithmetic system.

Logical axioms. The failure of Hilbert's programme led to a crisis in mathematical logic, but the critical rigor of mathematical knowledge did not suffer this crisis. It was precisely this critical rigor in the proving of Gödel's incompleteness theorem that led to the failure of Hilbert's programme. The failure of Hilbert's programme showed that in mathematical logic there are not only reason and critical rigor, but also logical axioms that are based on logical intuitions. Classical logic accepted the principle of excluded middle, according to which a statement is either true or false. This axiom coincides with the way logic is commonly used in mathematical reasoning. In fact, one very common type of argument in mathematics has always been the apagogical argument. An apagogical argument implicitly assumes the principal of excluded middle, in other words it implicitly assumes that A is either true or false. To argue that A is true using an apagogical argument, we assume that A is false and if we can demonstrate that this is a contradiction, then we have proved that A is false. The failure of Hilbert's programme questioned the excluded middle axiom. Critical

[14] DAVID HILBERT (1926), «Über das Unendliche», *Mathematische Annalen*, 95: 161-90. Lecture given Münster, 4 June 1925.

rigor does not require the excluded middle axiom to be accepted. The excluded middle axiom can be accepted and classical mathematical logic developed or it can be rejected and constructive mathematical logic can be developed whereby we can assert that A is true or false only when we can effectively prove that A is true or we can effectively prove that A is false. L. E. J. Brouwer (1881-1966) is considered to be the founder of intuitionism [15], which is a type of constructivism. Constructivism does not accept the excluded middle logical axiom.

4. Natural sciences and deductive reason

There is a gap between the formal world of mathematics and the real world of empirical science. This gap leaves a number of questions unanswered: What is the relationship between mathematics and the real world? What does a mathematician discover outside his mind when he has a mathematical intuition? Some mathematicians believe that the objects Hilbert called signs, and about which the mathematician felt direct intuition, really exist in a platonic world. But what is this platonic world? Where is this platonic world? What relationship is there between the platonic world and the real world? What is perceived by a mathematician when he senses that there is a number called one and another number called two and that the number two follows the number one in the same way as the number three follows the number two?

Mathematical theories help to represent empirical observations. The use of mathematics to formulate empirical knowledge is a characteristic of modern science. Mathematics' ability to formulate physical, biological and neuroscientific theories, etc. shows that mathematical intuition is not completely alien to empirical science; that the abstract objects in mathematics somehow exist in the real empirical world. We can only claim to fully understand and know about an empirical science when we are able to translate its statements into mathematical language. Mathematical statements and proofs written in mathematical language can be translated into any natural language. Mathematics is the most universal nucleus of natural languages. When the theory of relativity was mathematically formulated using Minkowski's mathematical theories [16] based on non-Euclidean geometry, this theory could be explained with the same precision in a mathematical language, regardless of the words, metaphors and statements about non-mathematical symbols used in the empirical and physical explanation of this theory.

Mathematical formulation and technological application. The mathematical formulation of theories enables them to be applied to technology. For example, the precise and mechanical nature of the mathematical explanation of the theory

[15] L. E. J. Brouwer, *On the significance of the principle of excluded middle in mathematics, especially in function theory,* 1923.

[16] S. Walter, *The non-Euclidean style of Minkowskian relativity. The Symbolic Universe,* J. Gray (ed.), Oxford University Press, 1999.

of relativity makes possible technological applications such as cathode rays, particle accelerators and GPS systems. Formal mathematical language enables descriptions of the mechanical processes used in machines and the instructions given so that machines can work mechanically to be written. Communication between man and machines is carried out using a formal mathematical language. Mathematical proof is a mechanical process itself.

Mathematical theories cannot fully represent empirical observations. Minkowski's mathematical theories are only mathematical theories and their meaning is purely mathematical. The theory of relativity explains the physical behaviour of empirical reality. Minkowski's theories do not in themselves explain the physical meaning of the theory of relativity. Empirical sciences use empirical information and mathematics does not have empirical information. There is a gap between the information dealt with in the formal abstract world of mathematics and the information from the real world of empirical science. Mathematical reasoning manipulates the information from mathematical intuitions formulated using mathematical axioms, whereas empirical science reasoning uses information from scientific hypotheses prompted by observations of empirical reality (Figure 5).

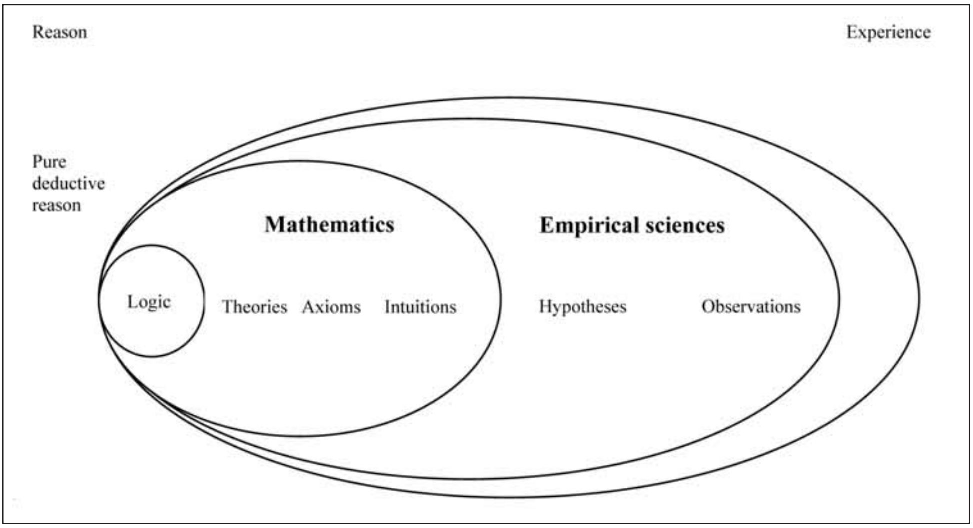

FIGURE 5

The difference between the intuition of an abstract mathematical object and a hypothetical induction using observations lies in the fact that the intuition of abstract mathematical objects cannot be contradicted by empirical observation. For example, Euclides' fifth axiom hypothesis can be accepted or rejected, but the decision to accept of reject it does not depend on empirical observations. The acceptance or not of quantum physics hypotheses however does depend on whether these hypotheses are contradicted or confirmed by new empirical observations. Empirical sciences are sciences open to empirical observation. They are based on hypotheses that explain observed facts, but which can be proved to be false by new observations.

The gap between the abstract world of mathematics and the empirical world implies that natural science observations can only be partially explained using a formal mathematical language. Empirical science needs to remain open to possible new observations and to new hypotheses about these observations.

Mathematical signs and scientific metaphors. In order to explain the gap between the world of mathematics and the empirical world, I will distinguish between the *formal signs* used by the syntax of mathematical logic and the *metaphorical symbols used by the languages of natural science* to represent observed objects. Formal signs are linguistic objects that represent mathematical objects. For example, +, <, 1, 2, 4, point, straight line, etc. are formal signs used to construct mathematical statements, such as 2 + 2 = 4, or geometrical statements, such as «two points determine a straight line». Mathematical statements are true or false in formal semantic models. For example, 2 + 2 = 4 is true in the structure of natural numbers. «Two points determine a straight line» will be true in different formal structures of Euclidean geometry.

Both mathematical statements and formal semantic models in mathematics are descriptive. But they do not describe empirical experiences, rather mathematical intuitions and mathematical axioms. The metaphorical symbols inherent in empirical sciences are, on the other hand, linguistic objects whose meaning is established using empirical models that represent the real world. For example, volume, energy, life, mind, etc. are metaphorical symbols. The statement $E = mc^2$ asserts, in the relativistic model, that volume and energy are interchangeable. What do metaphorical symbols describe? Basic inductions using experience.

If we use mathematical reasoning, we are not able to deduce empirical hypotheses from mathematical statements. Empirical hypotheses are induced using methodical observations. New methodical observations can lead us to confirm empirical hypotheses or to falsify them. When a scientific theory gets to the point of a mathematic formulation, it becomes truly interdisciplinary. Any scientist or scientific community can understand it.

5. Metaphysics, religion and deductive reason

It is normal for human beings to ask metaphysical questions. It is normal for human beings to ask why the world is understandable, why the real world exists, why goodness exists and it is normal for human beings to have answers to these questions. Just as it is normal for human beings to have a rational view of the world, it is also normal for human beings to have a complete and global metaphysical view of reality. Metaphysical views arise when someone, perhaps together with a school of thought or perhaps in a particular cultural context, becomes aware of their relationship with the world's global reality and with the fact of existence in this world and they look to find answers to the questions arising from these global realities.

Empirical science cannot answer metaphysical questions. Empirical science provides hypotheses about methodical observations of an aspect or a part of reality. Metaphysical statements must be linked in some way to each and every one of our perceptions about reality, including all our emotions and feelings. Metaphysical views and intuitions cannot be the logical conclusion of scientific arguments. Metaphysical statements express views about the world as a whole. The world as a whole and the world's possible relationship with God cannot be falsified by scientific experience.

But global metaphysical views and intuitions cannot be separated from other experiences, intuitions and reasoning of human knowledge. There is a global unity to human knowledge and metaphysical views and intuitions cannot break this global unity. No individual human experience or expression is completely alien to global views about the world and the universe. All mathematical intuitions, all scientific observations and all human experiences can be integrated or not into a global metaphysical view of the world. The history of human knowledge is full of conflicts and consistency between scientific and human views and intuitions and metaphysical and religious views of the world.

Metaphysical questions are inevitable. Asking metaphysical questions does not depend on one kind of culture or another, nor does it depend on the level of recognition achieved within a culture. Once we have stimulated our capacity to ask questions of ourselves, the question about the world's ultimate raison d'être is inevitable. If someone does not want to consider this question, they are already answering with their attitude. The answers we give to metaphysical questions constitute our view of the world. For example, for some people the world's intelligibleness is only partial; for these people, there are traces of consistency in the world, but the world is not always consistent. For Leibniz, world consistency and intelligibleness was somehow complete and God created the best world possible. Other people find enough of an answer to the question about the world's intelligibleness in the world itself, without referring to a God and creator transcending the world. Finally, for others there is no philosophical reason of a philosophical that can in any way explain the world's intelligibleness. This final stance is an agnostic view of metaphysics. This agnostic view recognises the question about the world's intelligibleness as valid, but does not believe that any other view beyond the natural sciences can help provide an answer to this question. Everyone can formulate metaphysical questions and the answers to these questions can vary from one person to another. A global view of the world can be agnostic or pantheistic when answers to the ultimate questions about the universe are asked in the same world, or it can be theist when answers to the ultimate existential question are looked for outside the universe, or it can be atheistic when the possibility of finding an answer outside the universe is rejected.

Metaphysical views are different from scientific views in terms of how they refer to the world as a whole. Scientific views always refer to a part or an aspect of the world. Scientific models represent an aspect of reality. They are like maps of reality and a map does not reproduce reality, rather it represents it. A metaphysical view however is a «way» of seeing reality as a whole. Empirical sciences are inevitably disciplines as they refer to an aspect of the world. Metaphysics is essentially trans-disciplinary, as it refers to the whole. Metaphysics questions reality as a whole. For example, the question about why reality exists is a metaphysical question not a physical one. Physics constructs models to interpret the laws that govern reality, but it does not hold the answers to questions about why these laws exist.

Metaphysical statements use symbols that refer to reality as a whole. Examples of metaphysical symbols include words and symbols that represent beings in general and the intelligibleness of beings in general. The symbols that represent beings in general do not refer to any particular object, as they refer to all objects, in the same way as when we discuss the intelligibleness of beings we do not refer to any object, rather to the ability all objects have of being understood. The difference between scientific metaphors and metaphysical symbols is that metaphysical symbols refer in some way to the whole reality, whereas scientific metaphors only refer to an aspect or part of reality. In the same way that each scientific community has its own metaphors and symbols to represent its view of reality, different philosophical communities also have their own symbols to represent their view of reality.

Metaphysical views are the basis upon which religions act and are developed. From the perspective of reason, which is the focus of this paper, all religions provide answers to metaphysical questions in some way or another. But metaphysical questions are not necessarily religious and the answers to these questions are not necessarily of a religious nature. A metaphysical question would be: Why is the world intelligible? This question is metaphysical because it does not ask the reasons or causes behind how we understand the world, rather why these reasons and causes exist. Metaphysical statements refer to views about the world as a whole. Theological statements are metaphysical statements that express views about the world in relation to God from religious foundations. Religious communities use theological symbols to explain how God reveals himself to human beings.

Man can understand religious symbols. Precisely because human beings are capable of using metaphysical symbols to express their metaphysical views and intuitions, they are also able to listen to religious words, statements and messages about the world as a whole and about God as creator of the universe (Figure 6). The Jewish, Christian and Islamic religions convey the message that God, creator of the universe, is present in the world through his own revelation.

Deductive reason and philosophical and religious communication. The different philosophical and theological formulations need to be conveyed between the groups as they refer to the same world. These formulations need to become cross-community. Structuring philosophical and religious formulations by pure deductive reason is the most solid foundation and point of contact not only for inter-disciplinary exchange between scientific communities and inter-cultural exchange between different human communities, but also for inter-religious exchange between different religious communities. Pure deductive reasoning is somehow an absolute value that transcends all aspects and levels of human knowledge, including metaphysical and religious knowledge. Metaphysical and theological arguments need to be able to span communities. Pure deductive reasoning is a kind of reasoning that can fully span communities and it forms the basis for inter-disciplinary, inter-cultural and inter-religious communication.

6. Metaphysics and religious revelation

The revelation of God (Figure 6) is at the centre of Christianity, Judaism and Islam. The revelation of God answers questions presented to man by metaphysics: the full meaning of life, the creation and existence of God and his revelation to the world. By reasoning about empirical experiences, the meaning of life and the need for religion, we do not deduce statements about the revelation of God. In the revelation of God, he is present in the world through his word. The world of God is linked to each and every thing in the world.

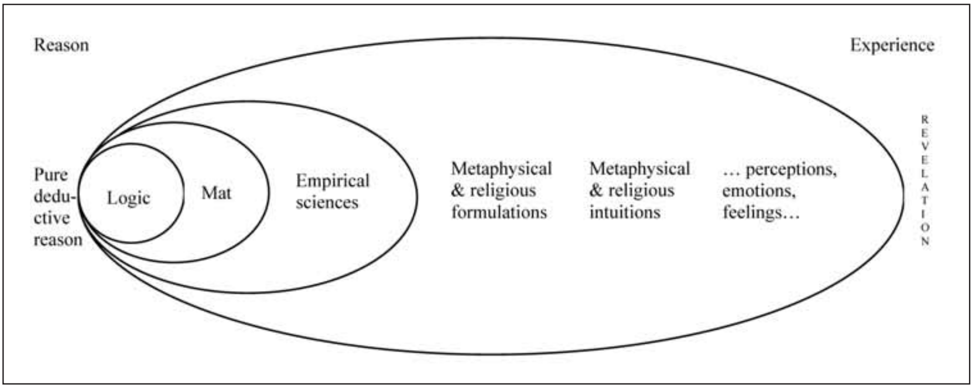

Figure 6

7. The paradoxical link between mathematics, reason and religion

Between mathematics, reason and metaphysics there is a paradoxical link. On the one hand, mathematical logic and metaphysics are two opposite dimensions of human knowledge with considerable differences between the two. On the other hand, there are some deeply-rooted similarities between metaphysics and the pure deductive reason and mathematical logic is the area of human knowledge with a clearest use of pure deductive reason.

Differences between mathematical logic and metaphysics:

1. Signs vs. Symbols: Mathematical logic employs signs with an objective and defined meaning. Metaphysical propositions use symbols whose meaning refers to reality as a whole and often to a transcendent God.
2. Conveyable vs. Indescribable: Whereas the propositions in mathematical logic can be translated into all natural languages and can be conveyed to anyone, many mystics have expressed their linguistic inability to convey their metaphysical experience about the ultimate sense and meaning of the world and also their relationship with God.
3. Contemplation vs. Control: Formal knowledge of the laws of nature allows reality to be controlled from a technological point of view. In metaphysical and religious views, understanding rests on a contemplative sense of mystery.
4. Definition vs. Holism: Mathematical and logical statements refer to a particular area of discussion. Metaphysical and religious propositions refer to reality as a whole.
5. Complexity vs. Simplicity: Knowledge of mathematical logic is specific, analytical and complex. Global metaphysical knowledge is man-made and simple.

Paradoxically, pure deductive reason is:

1. Simple, because it does not change
2. Holistic, because deductive reasoning is applicable to any kind of knowledge and is always the same in any place and at any time
3. Contemplative, because it does not depend on human activity.
4. Indescribable, because it is common to all languages.

Conclusion

At the beginning of this paper, I suggested that although mathematics and religion are at first glance very different and perhaps opposing realities, historically there are many links between the two. The relationship between mathematics and religion has often been opposing and conflicting and at other times, the relationship has been a question of mutual influence and even identification.

Throughout the paper, I have aimed to give information about the the nature of reason. I have tried to describe the presence of reason in mathematics and the presence of mathematics in all human thought, in particular through pure deductive reason. Pure deductive reason is present in some way in all human thinking. Pure deductive reason shares this universal presence with metaphysics and religion. They are two very different presences. Reason is actively present; structuring, organising and clarifying etc. Religion is present in a more contemplative way that I have called metaphysical. Metaphysics is understood to mean a radical and basic view of the world where everything matters, everything is included, looking for a rational answer to the question about reality and realities as a whole. Of all the sciences, it is mathematical logic that most clearly and precisely uses reason. There is a paradoxical relation between metaphysics and mathematical logic.

Pure deductive reasoning is somehow an absolute value that transcends all aspects and levels of human knowledge, including metaphysical and religious knowledge. Structuring philosophical and religious formulations by pure deductive reason is the most solid foundation and point of contact not only for inter-disciplinary exchange between scientific communities and inter-cultural exchange between different human communities, but also for inter-religious exchange between different religious communities. Metaphysical and theological arguments need to be able to span different cultural communities. Pure deductive reasoning is a kind of reasoning that can fully span communities and it forms the basis for inter-disciplinary, inter-cultural and inter-religious communication.

Metaphysics is the basis upon which religious views are developed. Metaphysical man is ready and able to receive religious revelations. But metaphysical man is neither just nor inevitably a cultured academic. Metaphysical man is open to questions about the meaning of life, the meaning of existence and the universe.

JAVIER LEACH

Universidad Complutense
Facultad de Informática
28040 Madrid
leach@sip.ucm.es

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