

# SOME RELATIONAL THEORIES ON THE STRUCTURE OF SPACE-TIME: PHYSICS, PHILOSOPHY, THEOLOGY

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**ABSTRACT:** We present some modern theories on the structure of space-time that can be classified as relational theories in the direction of Leibniz's ontology. In particular we summarize the Leibniz's position against Newton and Clarke on the nature of space-time, Penrose's model of spin networks, Heisenberg's fundamental equation for the unification of elementary particles, Finkelstein's space-time code, Weizsäcker's urs theory that unifies the postulates of quantum mechanics and the theory of relativity, Sorkin's causal sets, Markopoulou's quantum causal histories, Markopoulou and Smolin's causal spin foams, in the last three of which the principle of causality has been introduced. In order to analyze the nature of space-time, we consider three levels of knowledge —observational, theoretical and ontological— to which the different models can be ascribed. Following similar approach to the models mentioned in the first sections, we present our theoretical model of the structure of the space-time, some physical applications and the ontological interpretation of the model. In order to implement the above models with theological aspects we present in the last sections two theologians, Pannenberg and Torrance, who have made explicit analysis of the nature of space-time from a relational point of view. Following their expositions we have presented, after some epistemological presuppositions, the connection between the Creation and the rationality of God through the structure of space-time and the communication of God to the creatures in the Incarnation through the same structure.

**KEY WORDS:** space-time, relational theory, spin networks, urs theory, space-time code, causal sets, spin foams, quantum causal histories, causal cubic lattice, Creation, God rationality, Incarnation.

## *Teorías relacionales sobre la estructura del espacio-tiempo: física, filosofía, teología*

**RESUMEN:** Presentamos algunas teorías modernas sobre la estructura del espacio-tiempo que pueden ser clasificadas como relacionales siguiendo la dirección de la ontología de Leibniz. En particular resumimos la postura de Leibniz frente a la de Newton y Clarke sobre la naturaleza del espacio-tiempo, el modelo de los spin networks de Penrose, la ecuación fundamental de Heisenberg para la unificación de las partículas elementales, el código espacio-temporal de Finkelstein, la teoría de los urs de Weizsäcker, que unifica los postulados de la mecánica cuántica y la teoría de la relatividad, los conjuntos causales de Sorkin, las historias cuánticas causales de Markopoulou, los spin foams causales de Markopoulou y Smolin, donde en los tres últimos se ha introducido el principio de causalidad. Para analizar la naturaleza del espacio-tiempo consideramos tres niveles de conocimiento —observacional, teórico y ontológico— a los cuales podemos adscribir los modelos mencionados. Siguiendo las directrices de los modelos descritos en las primeras secciones, presentamos nuestro propio modelo teórico sobre la estructura del espacio-tiempo, algunas aplicaciones físicas y la interpretación ontológica del mismo. Para completar los modelos anteriores con aspectos teológicos presentamos en las últimas secciones dos teólogos, Pannenberg y Torrance, que han realizado análisis explícitos sobre la naturaleza del espacio-tiempo desde un punto de vista relacional. Siguiendo sus exposiciones hemos presentado, después de estudiar los presupuestos epistemológicos que subyacen, la conexión entre la Creación y la racionalidad de Dios a través de la estructura del espacio-tiempo, y la comunicación de Dios a las criaturas en la Encarnación a través de la misma estructura.

**PALABRAS CLAVE:** espacio-tiempo, teoría relacional, spin networks, teoría de los urs, conjuntos causales, causal spin foams, historias causales cuánticas, retículos cúbicos causales, Creación, racionalidad de Dios, Encarnación.

## 1. INTRODUCTION

My approach to the nature of space-time can be considered a relational theory, following the critical position of Leibniz (section 2) against the absolutist theory of Newton. In order to understand better my ontological interpretation I have presented some modern authors who have helped me to clarify the epistemological presuppositions as well as the ontological background that they have worked out. They are:

- i) the spin networks of Penrose (section 3),
- ii) the unified theory of elementary particles of Dürr and Heisenberg (section 4),
- iii) the space-time code of Finkelstein (section 5),
- iv) the theory of simple alternatives (urs) of Weizsäcker (section 6),
- v) the causal sets of Sorkin (section 7),
- vi) the quantum causal histories of Markopoulou (section 8),
- vii) the causal spin foams of Markopoulou and Smolin (section 9).

In my opinion these models belong to the relational theories of space-time, and their authors have tried the unification of quantum mechanics with the theory of relativity, in such a way that from the principles of Quantum Mechanics the structure of space-time is derived, and the basis for the theory of relativity emerges.

For the presentation of our model we start with the epistemological presuppositions necessary to locate the concepts of space and time (section 10); it turns out that these are derived concepts. Then we present our model in two different examples, the cubic and the hyperbolic lattice (section 11). To make contact with physics we review some papers where we have translated the continuous language of physics into the discrete one (section 12). Finally we describe the ontological background (underlying the theoretical level) made out of interactions of elementary beings, from which the concepts of space-time are derived (section 13).

In the Last sections we have implemented the relational theories of space-time with some theological nterpretations given by Pannenberg (section 14) and Torrance (section 15).

## 2. LEIBNIZ'S RELATIONAL THEORY OF TIME AND SPACE

In his correspondence with Clarke, Leibniz defended his position of a relational theory against Newton. Time is the order of those points not existing simultaneously and one is the ratio of the other. Space is the order of points that exist simultaneously and are connected by mutual interactions. Space is nothing more than the set of all points and their relations [1]. Position is that relation what the same is in different moments for different existing points and their coexisting relations with some

particular points coincide completely [2]. A point changes its position if it changes its relations from some points to different ones. Motion is the change of different positions in time. Similar definitions of time and space have been given by Leibniz in his mathematical article «Metaphysical principles of mathematics» [3], with the added concepts of duration and extension as magnitudes of time and space.

### 2.1. *The ideality of space*

The definitions given above have to be considered in some epistemological presuppositions. As Earman says:

«There are passages from the 1680s in which Leibniz specifically refers to space and time as well-founded phenomena. Such passages only seem to compound the puzzle of the ideality thesis. The puzzle is resolved by noting that such passages disappear in the 1690s when Leibniz begins to make use of a trichotomy consisting of the monads, well-founded phenomena, and a third realm consisting of variously labeled 'ideal', 'neutral' and 'imaginary'. It is to this third category that space and time are confined in Leibniz's later writings» [4].

According to this interpretation Leibniz's concepts of space does not correspond neither to the metaphysical level nor to the phenomenological level of observation and measurement, but to an intermediate level of knowledge that Leibniz calls ideal, and we call theoretical level.

### 2.2. *The monads*

In the ontological level Leibniz presents his Monadology. The monads correspond to the geometrical points in the theoretical (ideal) level. A monad is the metaphysical unity of a body (matter) with its entelechy (substantial form). Monads are the constituents of physical bodies, living organisms and human beings.

In the critical edition of the Monadology, Velarde explain the activities (action-passion) of monads:

«The internal principle is the force, as internal power of expansion, that generates a system (or state) of specific and internal qualities in each monad, called perception; and the action of internal principle that produces the change (or transit) from one to another state (from one to another perception) is called appetite (or appetite). Perception and appetite are thus metaphysical notions. Perception explains the monad as far as it is determined from the world (passiv aspect), the appetite, by the contrary, presents it to us in its activ aspect, in the motion (tendency-impetus) from one to another perception. All the monads have a perceptive relation with the universe... perception is nothing more but a plurality of relations of each monad with the rest» [5] (traslation of M. L.).

In the next sections we are going to present some models of contemporary physicists who have followed the relational theory given by Leibniz about the

nature of space-time. We will present also the epistemological presuppositions underlying their models. In their effort to justify their position they have reviewed the general principles that are involved in the unification of general relativity and quantum mechanics.

In order to achieve this goal it is unavoidable to understand better the nature of space-time because it is a common «arena» for both theories.

### 3. PENROSE'S SPIN NETWORKS

We can consider Penrose's ideas of space and time very similar to the relational theory, because the concepts of space and time are derived from the set of interrelations among fundamental entities.

#### 3.1. *Motivation*

Penrose considers the use of continuum in physics of mathematical utility rather than of essential physical necessity:

«I wish merely to point out the lack of firm foundation for assigning a physical reality to the conventional continuum concept» [6].

The alternative proposed by Penrose is to derive the concepts of space and time from some combinatorial principles:

«My idea is to try to reformulate physical laws so that they may be expressed entirely in terms of quantities which are discrete according to quantum physics» [7].

In Penrose's model, quantum physics offers a collection of simple elements which are discrete, out of which the space and time can be derived.

#### 3.2. *The Model*

For completeness we summarize the main tracks of the theory of spin networks that can be founded in the primitive paper of Penrose [8]. The starting point is the total angular momentum of some fundamental unit (elementary particle or physical system). A set of units are acting among themselves following the quantum rules of total angular momentum. In order to obtain a direction for some unit we need a large angular momentum such that the direction can be represented by the projection of the total spin  $j$  on the  $m$ -component of the system in the direction of the  $z$ -axis. The value of the  $m$ -component of the unit can be obtained by combinatorial process between two large units, that can be interpreted like the angle between the relative directions of both units.

The above picture is responsible for the emergency of eucliden geometry out of networks of units. If we want to implement this picture in the relativistic case, we have to introduce orbital components that require position of the units. Again,

when two units are acting, a mutual displacement is emergent giving rise to the concept of position. So in the relativistic case a real 4-dimensional space-time is emergent from the interrelations of two or more elementary units.

### 3.3. *The background and the geometrical space*

Finally, the ontological status of the model is based in the interrelations between objects. As Penrose says:

«My model works with objects and the interrelations between objects. An object is thus “locate” either directionally or positionally in terms of its relations with other objects. One does not really need a space to begin with. The notion of space comes out as a convenience at the end» [9].

On this model Penrose makes a very important distinction between the auxiliary space (the background space) necessary to define total angular momentum in terms of spherical coordinates and the geometrical space that results from the angle between two spin directions. The last space is the real one and corresponds to the observed physical space [10].

## 4. HEISENBERG'S FUNDAMENTAL EQUATION

Although Heisenberg was one of the founders of Quantum Mechanics he never was satisfied with the orthodox interpretation and tried with some of his students and collaborators to modify the underlying ontology of Quantum principles. According to Weizsäcker, who elaborated his dissertation with Heisenberg, the epistemological position of this is neither realistic (the realist thinks he knows a priori what reality means) nor positivistic (the positivist thinks he knows a priori what experience means) and is closer to Kantian philosophy by which certain elements of the theory are taken to be preconditions of experience. In any sense, Heisenberg rejected the position taken by these three epistemologists because they presuppose the traditional opposition of observer and observed.

«When I was studying with him in Leipzig around 1930 he was already pondering on possible explanations of pure numbers like Sommerfeld's fine structure constants. According to this epistemological view there would have to be a new theory of elementary objects beyond general quantum theory... His speculation of a fundamental length and even the introduction of the S-matrix belonged to this approach. In the meantime, reasons have become strong for the view that elementary particle theory is to be a perfect normal quantum theory which only limits the list of possible physical systems by additional axioms» [11].

### 4.1. *Heisenberg's unified theory of elementary particles*

The central assumption about additional axioms necessary to unify elementary particles from a quantum field theory is the invariance under some symmetry group. The model consists on a unique linear spinor field equation for a 4-

component local Weyl-spinor-isospinor field operator which obeys non-canonical anticommutation relations. These spinors-isospinors fields are vector components of the representation of the Poincaré group and of the  $U1 \times SU2$  group which (except for  $U1$ ) all occur in the generalized form of gauge symmetries of second kind. Dürr has been working with Heisenberg in the detailed application of this model to the exuberant world of elementary particles.

The question that Dürr put forward on Heisenberg *Weltanschauung* is whether or not there was another conceptually more basic level beneath our level of description that corresponds to the classification of elementary particles.

## 5. FINKELSTEIN'S SPACE-TIME CODE

Dürr claims that Heisenberg was convinced of this hypothesis, and has tried to make connection of the more fundamental ideas of Weizsäcker's *urs* or Finkelstein's space-time code with Heisenberg's unifying theory, in such a way that the continuous non linear field equation of Heisenberg is a limiting case of discrete models of Weizsäcker or Finkelstein [12].

### 5.1. *Finkelstein's process theory*

According to Finkelstein the world is represented by a network of quantum processes which, in one version of his work [13], is built from «tetrads» as the only basic connected elements forming a structure with a checker board topology. The checker board constitutes the underlying structure of space-time manifold. This discrete structure can be considered the «arena» where the displacements and interactions of elementary particles take place. In particular Dürr has shown that the non linear spinor Heisenberg's equation without its isospin degrees of freedom together with the hermitian conjugate equation (combined to a single non linear field equation for a 4-component hermitian Majorana spinor field) corresponds precisely to Finkelstein tetrad, in such a way that proceeding a discrete step in the checker board a new tetrad appears giving rise to the propagation in the network [14].

Finkelstein has developed his program for the ontology of processes starting from the concept of monads, that reminds us of the Leibniz conception of material points out of which the structure of space and time is created. Finkelstein's ideas do not presuppose the existence of space and time, a similar position taken by Penrose [15].

## 6. WEIZSÄCKER'S UR HYPOTHESIS

According to Dürr, the unification of elementary particles proposed by Heisenberg was inspired in Weizsäcker's theory of simple alternatives. This

author, who tried first with Heisenberg a reconstruction of Quantum Mechanics, worked later with his group in Starnberg a new foundation of Quantum Mechanics [16].

### 6.1. *Postulates for the Basic Structure of Quantum Mechanics*

Weizsäcker new conception is derived from empirical and philosophical reflections on physical objects, that lead to some abstract, concrete and full Quantum Theories [17]. The abstract theory is constructed from Hilbert space, probability metric, rules of composition and dynamical laws. The concrete theory is constituted from real facts that can be reduced to simple alternatives, experiments yes/no, that are called *urs*, with the following properties: i) every experimental result can be reduced to a finite number of alternatives, ii) the number of possible alternatives is unlimited, iii) there are objects with only one alternative.

It corresponds to the full Quantum Theory to unify the abstract and concrete theory through a set of postulates, namely: i) Postulate of separable alternatives: there are alternatives whose states are separable from nearby all other states. Therefore we need only a finite number of alternatives to determine an object completely. ii) Postulate of indeterminacy: if a probability vector of the outcome of the possible alternative is defined, this must be a continuous function, in order that some state not belonging to the given alternative can be produced.

These postulates satisfy two conditions, which are accepted as preconditions of our experience: i) there is an actual infinity of future possibilities, and ii) no recourse to an actual infinity of facts is needed. In other words, the actual number of the simple alternatives in the universe is finite, but the number of possible alternatives in the future is infinite and they are governed by laws of probability [18].

### 6.2. *The epistemology of ur-hypothesis*

The goal of Weizsäcker's reconstruction of quantum theory was to unify Quantum Mechanics with Theory of Relativity in such a way that the fundamental theory corresponds to the principles of Quantum Mechanics and the derived concepts correspond to the Theory of Relativity. In particular, if we accept the *Ur-hypothesis* as the fundamental one, the structure of space and time is a consequence of the former. The Hilbert space of the *urs* consists of the complex vector with two components (yes-no decisions). Its symmetry group is  $SU(2)$ , but this group is isomorphic to the group of rotations in the real three dimensional space, and this is the explanation why ordinary space is three-dimensional [19].

The epistemological status of *Ur-hypothesis* can be understood if we locate it in the theoretical level between the empirical and the ontological one. In the theoretical level quantum physics and the Theory of Relativity are derived from the same principles. But can we ascribe a more fundamental level, an ontological level, to the *Ur-hypothesis* in such a way that there exist a correspondence between the former and the later as seen by a philosopher and a physicist?

We recall the comment of Görnitz and Ischebeck on Weizsäcker's Weltanschauung:

«The overwhelming success of science in the material world left no place for spirit in science. This process could be reversed by a quantum theory based on the foundations which Weizsäcker has given, where matter and energy are united with information. On this basis it is possible that consciousness becomes a genuine part of natural science» [20].

Similar approach to the philosophy of physics contained in the Ur-hypothesis is given by Lyre in his article on Weizsäcker's Reconstruction of Physics. If one takes seriously Aristotle's *Metaphysics* all the substance in the world are composed out of matter (*materia prima* or *hyle*) and forms (*forma substantialis* or *eidos*). This last component can be identified with information and this is precisely what Weizsäcker adscribes to the *urs*: a bit of information what it is obtained by the yes-no experiment. The *Ur* is reduced to the *eidos* in the ontological level [21].

## 7. SORKIN'S CAUSAL SETS

The next two sections will deal with some variations of spin networks that take into account the causal relations among the elements of the discrete sets of events. In both cases, Sorkin and Markopoulou, introduced the hypothesis that there is a discrete reality underlying the continuous space. This reality or new substance is a causal set. Historically Sorkin claims that his ideas are rooted in Riemann's conception of discrete manifold, in which the principle of its metric relationships is already contained in the concept of the manifold itself. According to Sorkin:

«The causal set is, of course, meant to be the deep structure of space-time... the space-time cease to exist on sufficiently small scales and is superseded by an ordered discrete structure to which the continuum is only a coarse-grained, macroscopic approximation» [22].

Sorkin's decision to accept causal sets was a reaction to the operationalism view of science, by which all the knowledge of nature are reduced to the set of operations by which we observe the experimental data. He accepted the ontological view that causal set is a real substratum, existing independently of any experimental activity of our part, and the elements of a causal set are real, and the notions of length and time emerge from relations among some fundamental entities [23].

### 7.1. *A causal set and its embedding*

The discrete structure of space proposed by Riemann in 1854 was elaborated later by Robb [24] in 1914, where he proved that the geometry of 4-dimensional flat space-time can be recovered from nothing more than the underlying points set and the order relation among points. Also Reichenbach [25] stressed the

same fact and Finkelstein proposed in 1969 the original model of a causal set [26]. As a mathematical structure, a causal set is a locally finite ordered set, i.e. a set  $C$  endowed with a binary relation  $<$  possessing the following three properties:

- i) Transitivity:  $(\forall x, y, z \in C) (x < y < z \Rightarrow x < z)$
- ii) Irreflexivity:  $(\forall x \in C) (x \not< x)$
- iii) Local finiteness:  $(\forall x, z \in C) (\text{card} \{y \in C \mid x < y < z\} < \infty)$ .

Property ii) implies the absence of cycles and property iii) is a formal way of saying that a causal set is discrete.

In order to compare the discrete causal set with the continuum space-time, Sorkin and collaborators introduce an embedding in such a way that i) causal relations among the points in the discrete are preserved in the continuous and ii) the embedded points are distributed uniformly with unit density. If these conditions are satisfied one can decompose the continuum manifold in elementary volumes such that to each one correspond one point, and in this way the Criterion of Riemann is fulfilled, that measuring is counting [27].

## 8. MARKOPOULOU'S QUANTUM CAUSAL HISTORIES

As we mention in the last section, Markopoulou makes the hypothesis that the underlying reality at the Planck's scale is discrete and it can be described by spin networks endowed with causal relations. This is appropriate to the canonical quantization of general relativity, in the sense of loop quantum gravity. Loop quantum gravity gives an exact microscopic description of spatial quantum geometry in term of basic states called spin networks. The dynamics is expressed in path integrals defined in terms of amplitudes for local moves along the spin networks. The basic operators of the theory (area and volume) are quantized. This construction, according to Markopoulou, suggests that at Planck scale geometry is discrete. Besides that, the theory is background independent, it does not live in a preexisting space-time. As Smolin claims:

«At the end what is most satisfying about the picture of space given by loop quantum gravity is that it is completely relational. The spin networks do not live in space; their structure generates space. And they are nothing but a structure of relations, governed by how the edges are tied together at the nodes» [28].

One of the main issues of loop quantum gravity is the problem of the low energy limit. From the fundamental combinatorial dynamics at low energy have to emerge the classical space-time and the dynamics of general relativity [29].

### 8.1. *Quantum causal histories*

Markopoulou has summarized several features in common with models of microscopic structure of space-time. They are the following [30]:

- i) At energies close to the Plank scale the Universe is discrete.
- ii) Causality still persists; the Universe is described by the rules of causal sets presented by Sorkin *et al.*
- iii) Quantum theory is still valid at this level.
- iv) The model should be background independent.

These presuppositions are taken in account by Markopoulou to construct the quantum causal histories; for completeness we sketch them in the form of causal sets plus quantum operators [31].

*Causal set*: partially ordered set, locally finite, with preceding relation  $\{C, <\}$ .

Causal past:  $\{r \mid r < p, r \in C\} \equiv P(p)$ .

Causal future:  $\{q \mid p < q, q \in C\} \equiv F(p)$ .

Set  $\underline{a}$ , is a complete past of  $p$  if every event in  $P(p)$  is related to  $\underline{a}$ .

Set  $\underline{b}$ , is a complete future of  $p$  if every event in  $F(p)$  is related to  $\underline{b}$ . Two sets  $\underline{a}$  and  $\underline{b}$  are a complete pair if  $\underline{a}$  is a complete past of  $\underline{b}$  is a complete future of  $\underline{a}$ .

*Quantum causal sets*: attach a Hilbert space to each event of a causal set representing elementary systems.

*Quantum causal histories*: the evolution of a Quantum causal set is implemented by unitary operator between Hilbert spaces of a complete pair.

*Quantum spin networks*: repeated applications of local moves takes one spin network into another.

Therefore, in quantum causal histories, with the Hilbert spaces on the events and the operators on the causal relations, the quantum evolution strictly respects the underlying causal set. The ontological background of the space-time—the quantum space-time— consists on a very large set of open systems joined by quantum operations, where unitary evolution arises only for a complete pair [32].

## 9. MARKOPOULOU AND SMOLIN'S CAUSAL SPIN FOAMS

Penrose's spin networks can be considered as a graph with edges labelled by irreducible representations of the group  $SU(2)$  and vertices labelled by intertwiners satisfying the rules of angular momenta. Similarly a spin foam is a 2-dimensional complex with faces labelled by irreducible representations of a group, generally the group  $SO(4)$  or  $SO(3,1)$ , and edges labelled by intertwiners. (The 2-dimensional complex can be considered the 2-dimensional dual graph coming from the triangulation of a 4-dimensional manifold where to each 4-simplex corresponds a vertex and to each tetrahedron corresponds an edge) [33].

### 9.1. Spin foam models

With the help of spin foams one constructs spin foam models for quantum gravity that are intrinsically discrete and are supposed to go in the low energy

limit to the general relativity field equations and the continuous space-time manifolds.

Several spin foam models have been proposed [34] such as Lorentian path integrals, string networks or topological quantum field theory. The most elaborated of them and thoroughly studied is the Barrett-Crane spin foam model [35].

### 9.2. *Causal spin foam models*

Using the kinematical setting of partition function for spin foams and the assumption of a micro-local causal structure (encoded in the orientation of spin networks) Markopoulou and Smolin define a general class of causal spin foam model for quantum gravity [36]. The elementary transition amplitude for an initial spin network to another spin network is defined by a set of combinatorial rules.

Levine and Oriti have shown [37] that the Barrett-Crane model is the first non trivial example of a causal spin foam model, and that it represents a link between several areas or research, like canonical loop quantum gravity, sum over histories formulation, causal sets and dynamical triangulation.

### 9.3. *Background independent models*

Spin foam models have very important property. They are background independent quantum gravity models. They don't live in a pre-existing universe. They start with an underlying Planck scale quantum system with no reference to spatial temporal geometry. The geometry is defined intrinsically using subsystems and their relations. (Both quantum geometry and gravity emerges as a low energy continuous limit.) In particular, spatial and temporal distances have to be defined internally by observers inside the system.

Markopoulou has developed a lengthy discussion on the topic [38] and has given several definitions, the first of which is: «A theory is background independent if its basic quantities and concepts do not presuppose the existence of a given background space-time metric».

This is consistent with the relational principle by which the metric has to be defined internally, and in the case of a discrete manifold by counting the elements of the same manifold as Riemann claimed in his Inaugural Dissertation of 1854.

## 10. OUR MODEL: EPISTEMOLOGICAL PRESUPPOSITIONS

We can summarize the epistemological position of the authors mentioned in section 2 to 8 by three levels of human knowledge in the comprehension of the physical world. It will help to understand my own position in the interpretation of space-time [39]:

*Level 1:* Physical magnitudes, such as distance, interval, mass, event, force, and so on, that are given by our sensations and perceptions.

*Level 2:* Theoretical models, which are the generalization of metrical properties given by measurements and numerical relations among them.

*Level 3:* Fundamental concepts, representing the ontological properties of physical world given by our consciousness in an attempt to know the reality.

There must be some connections between the three levels. In Quantum Mechanics the theoretical models of microphysics in level 2 are related to observable magnitudes in level 1 by correspondence laws.

If we accept level 3 should be connected to level 2, an immediate question is to ask about the justification of the rules governing the construction of theoretical systems. It would be ridiculous to postulate them as game rules. They must be grounded in properties of the world they want to describe. For instance the unification of Quantum Mechanics and the theory of Relativity should be made in level 2 where they belong to, but the underlying ontological concepts should be taken from level 3.

We can now raise the following question: in level 1 we find primitive and derived concepts. According to philosophy of science it is almost impossible to decide whether some simple observable is primitive or not, because it depends on the type of experiment we have used to define it. Once we have decided the primitive concepts of a theory, the rest are derived concepts. The question to put forward: are the concepts of space and time primitive or derived concepts?

In absolute theories, space is a container where the particles are moving.

Time is also a separated entity with respect to which the motion takes place. Therefore space and time are primitive concepts and can be thought of in the absence of particles.

In relational theories, space and time consist on the set of relations of some fundamental objects. Obviously in this case, the concepts of space and time are derived. As Markopoulou explains: «Space-time geometry is a derivative concept and only applies in an approximate emergent level» [40].

This is a consequence of the relational character by which «spatial and temporal distances are to be defined internally by observers inside the system» [40].

## 11. A RELATIONAL THEORY OF SPACE-TIME: THE CAUSAL CUBIC LATTICE

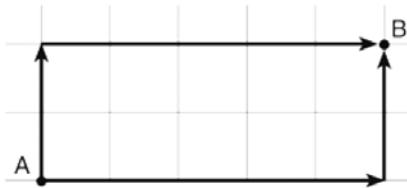
Following the assumption of the last section now we give an explicit construction of a formal structure of space-time, without the recourse to intuition. We can think of a set of fundamental objects acting among themselves, giving rise to a network of relations. These relations do not presuppose some space. The objects are nowhere if we consider them as elements of the physical world in level 2. In order to be specific we take as a naive network a three-dimensional cubic lattice. Obviously the network can be taken with different structure, such as, triangular, quasiperiodic or random lattices. In order to make connection with the euclidean geometry we take, for simplicity, an infinite set of interacting points in the relation 1 to 4, where one point is connected with no more and no

less than four. The set of all relations form a two-dimensional lattice, in which we can define:

A *path* is the connection between two different points, say, A and B, through points that are pairwise neighbours [41].

The *length* of a path is the numbers of points contained in the path, including the first and the last one.

A *minimal path* is a path with minimal length (in the picture the two paths between A and B are minimal). Between two point there can be different minimal paths.



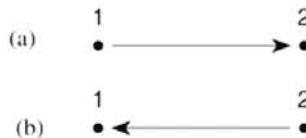
A *principal straight* line is a indefinite set of points in the lattice, such that each of them is contiguous to other two, and the minimal path between two arbitrary points of this line is always unique.

*Theorem 1.* Through a point of a 2-dimensional square lattice pass only two different principal straight lines (they are called *orthogonal straight lines*).

*Theorem 2.* Two principal straight lines that are not orthogonal have all the points either in common or separated (in the last case they are called *parallel straight lines*).

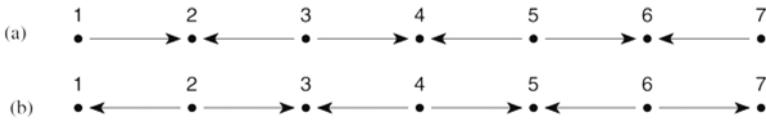
From these two theorem we can define Cartesian (discrete) coordinates and an Euclidean space where the postulates of Hilbert can be applied (with the exception of the axioms of continuity). This structure of 2-dimensional space can be easily generalized to 3-dimensional cubic lattice. As we mentioned, those assumptions for the structure of space are given in level 2, but it corresponds to the properties of physical space described in level 1 by our sensations.

In order to introduce the relation that correspond to time we start with only two fundamental objects acting among themselves:



In (a), 1 is acting on 2, and in (b) 2 is acting on 1. But the action of 1 on 2 is supposed to be a necessary condition for the action of 2 on 1, and similarly the action of 2 on 1 is supposed to be a necessary condition for a new action of 1 on 2. Thus we can think of a chain of mutual interactions arranged in a

series of necessary conditions. This picture has to be enlarged for the whole lattice.



In (a), 1 is acting on 2, 3 is acting on 2 and 4, 5 is acting on 4 and 6, 7 is acting on 6. In (b), 2 is acting on 1 and 3, 4 is acting on 3 and 5, 6 is acting on 5 and 7.

We postulate that the actions of (a) are necessary conditions for the actions of (b) and the actions of (b) are necessary conditions for a further action of type (a) and so on.

Now take a network of objects acting in the relation 1 to 4.

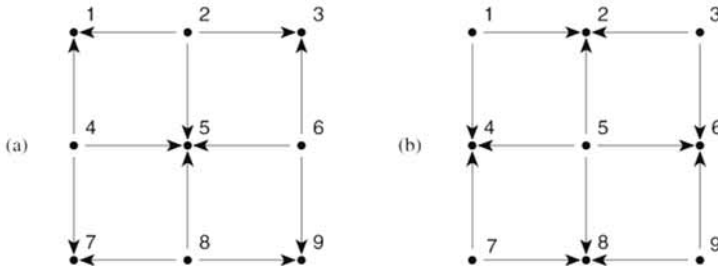


Fig. 1

Fig. 2

In (a), 2 is acting on 1, 3, 5; 4 is acting on 1, 5, 7; 6 is acting on 3, 5, 9; 8 is acting on 5,7,9. In (b), 1 is acting on 2 and 4; 3 is acting on 2 and 6; 5 is acting on 2, 4, 6, 8; 7 is acting on 4 and 8; 9 is acting on 6 and 8. As before we postulate that the actions of (a) be necessary conditions for the actions of (b) and so on. These logical properties of interactions belong to level 2 and do not presuppose the concept of time, but they can be put in correspondence with the physical properties of time given in level 1.

Similar causal relations can be assumed in the hyperbolic lattice [42].

## 12. PHYSICAL AND PHILOSOPHICAL IMPLICATIONS OF THE MODEL

The assumption of our model of space-time implies some physical consequences for the classical as well for the quantum physics:

- i) The space-time is discrete, therefore the physical laws are written in the language of finite differences. The solutions have to be described by continuous functions of discrete variable. We present some particular example in [43].
- ii) The symmetry of the model, in case of a Minkowski hypercubic lattice,

- is still Poincaré group, although one has to select those integral transformations that keep the lattice invariant [44].
- iii) Lattice gauge theories are not only a mathematical tool but a realistic theory, because they correspond to the underlying discrete structure of space-time [45].
  - iv) In General Relativity Riemannian manifolds have to be substituted by discrete graphs where geometrical magnitudes like metric tensor, curvature, have to be calculated by intrinsic properties of the graph [46].

In order to study the topology of a graph we embed it on a continuous manifold. Some quantum gravity models are based on this technique: the underlying space-time is discrete, but its embedding is continuous, where field representations are attached [47].

In our model from the data of our observations we have constructed theoretical models with the help of which we can give explanations and make predictions.

We have substituted the physical structure of space-time by some network of interrelations among fundamental entities from which the concept of space-time emerges.

In order to deepen in the nature of space-time we suppose there is an ontological level (level 3) where the physical properties of level 2 are interpreted with the metaphysical principles of the material objects.

In other models we have reviewed in the first sections we find some epistemological presuppositions to better understand their model. In Penrose's conception there are three «worlds» (inspired in Popper's philosophy) that correspond to the physical, the mental, and Platonic mathematical objects, such that all of them are cyclically and misteriously connected [48]. The difference with our epistemological scheme is that the Platonic or mathematical world, which would correspond to our ontological level, is lying in an ideal world outside of the physical one.

In the causal set model Sorkin presupposes that a physical theory passes through three stages: an initial stage in which a particular «substance» or type of matter presents itself in a characteristic group of phenomena; a second stage in which the new substance is clearly discerned in relation to the phenomena; and a final stage in which the comprehensive dynamics characterizing this substance is understood [49]. Sorkin claims that the new substance underlying space-time is a causal set, and he is convinced, contrary to operationalist ontology, that the elements of causal sets are real [50]. Therefore in Sorkin's model we find three level of knowledge: phenomena, physical theory and reality.

In the causal spin foam model and in the quantum causal histories there is a combination of causal sets with quantum mechanics in such a way that in the first model the non existence of the wave function of the universe imposes causality to the quantum subsystems, and in the second one the Hilbert spaces are attached to the events of the causal set. We find these two models very similar to our model with respect to the first and second level but they lack some ontological interpretation that we are going to present in the next section for our model.

### 13. AN ONTOLOGICAL INTERPRETATION OF THE NATURE OF SPACE-TIME

In our epistemological presuppositions for the interpretation of space-time, we have postulated the level 3 as the ontological background of the theoretical models of level 2. In a relational theory of the nature of space-time, the concept of substance should be adscribed to the fundamental entities —monads, urs, units, events— the interaction of which give rise to the set of relations responsible of the structure of space-time.

Is it possible to make some Ansatz about the nature of these fundamental objects? If we take the extension as the first property of matter, as Descartes has claimed, space and time should be considered necessary at the beginning of a fundamental theory. We prefer the point of view that the most essential property of material objects is the capacity of producing effects in other objects, which was identified by Leibniz with the concept of force [51].

There is a causal relation between the force and the effect (the principle of external causality in Aristotelian philosophy). The set of all causal relations among the fundamental objects can be taken as the ontological background in level 3 for the relational theories of space-time, such as Penrose's spin networks, Sorkin's causal sets, Markopoulou's causal quantum histories.

But still the picture is not complete. When the principle of causality is applied in classical mechanics or in the theory of special or general relativity it is supposed to follow the law of determinism: Given some mechanical system under particular initial conditions the same forces will produce always the same effects.

If we want to implement the principle of causality with quantum effects, as in the causal spin foams, we have to introduce the probability laws in the production cause-effect, as required by the postulates of quantum mechanics. Coming back to the level 3 the ontology of material objects is characterized not only by the principle of causality but also by the laws of probability [52].

#### 13.1. *Analogies and differences*

We have presented the ontological status of our model and now we want to compair it with others we have mentioned before. All of them start from some fundamental entities —monads, processes, units, urs, events— the interrelations of which produce a network responsible for the emergency of space-time.

The set of these elementary entities is locally finite, a condition necessary for the discreteness of space-time. It means that each elementary entity is individually separated from the rest [53].

Causality is a fundamental property for all entities and is responsible of the interactions among them. All the causal space-time networks can be reduced to the evolution (in discrete time) of several causal networks for different discrete time values [54]. In particular the evolution in time of the causal cubic lattice (section 11) can be reduced to two causal sets (figures 1 and 2) for two different discrete times.

But we have also detected some discrepancies. First of all in their models there is an identification of epistemological level 2 and 3, or, even more, they don't mention the ontological level because the elementary entities are reduced to some physical effects—simple alternatives, yes/no experiments, combinations of two angular mementa, events— where no mention of the ontological status is given.

Secondly, the lack of substantive character of elementary entities makes very difficult to predict the situation of these entities after the causal effect has taken place. Do they disappear? Are they transformed in other entities? In our model the action of some elementary being produces some effect in other being, but both beings persist in their existence. As a consequence the underlying network elementary beings persist in time.

A third difference is concerned with the embedding. In other models causal sets are embedded in some continuous manifold, such that we can talk about some elementary length between two different events connected by some causal action. In our model there is no elementary length because the distance between two causal interactions is reduced to the process of counting [55].

#### 14. PANNENBERG AND THE NATURE OF SPACE-TIME

We arrive to the last part of our work, the theological background of the nature of space-time. We have chosen two theologians who accept christian religion, namely Wolfhart Pannenberg and Thomas Torrance. They have explicitly work out some presentation of the nature of the space-time and its connection with the eternity of God, and also both of them are in favour of a relational theory of space-time, that is considered more adapted to created beings, although they don't reject the absolutist theory which is more consonant with the omnipresence of the Creator.

Coming to the first theologian W. Pannenberg we have selected several paragraphs from his book «Systematic Theology» [56], vol. 2, chapter VII, where he dedicates a complete section to the theme «Space and time as aspects of the Spirit's working». In this section he defines the space-time as a set of relations without recourse to another activity of the creatures. In addition he finds the ground for these relations in the self distinction of the three persons and the interaction with created beings. Finally Pannenberg presents the different stages of time and its connection with eternity: past, present and future and the dynamics of the Spirit of God.

##### 14.1. *Theological background*

In the creation of finite beings, God creates a multiplicity of places, and therefore a multiplicity of partial spaces, each separated from the rest because of the limited finitude. But even before the creation, we can find a multiplicity in God Himself:

the multiplicity of three Persons in God. We can think of mutual relations among the three Persons similar to the relations among different creatures, but with one very important difference: the distinction among the three Persons is not complete, each Person is distinct from the other one but at the same time is united with the other one. This self differentiation of the Father with respect to the Son and equally of the Son with respect to the Father, can not be applied to the creatures, although they are beloved by the Father as an expression of the love of the Father with the Son. The space of the creatures is based in the distinction among them and the rest of the finite beings and with God, and, consequently in the relations among all of them. «From this standpoint, space is the epitome of relations between divides spaces, between points of space» [57]. The last sentence is proposed by Pannenberg to avoid the following objection: this divided space is divided into smaller spaces and one can go in infinity like the points.

The theological interest for the relational theory of space and time is connected with the creation of finite beings. S. Agustín proposed an argument against the hypothesis of indefinite spaces (*infinita spatia locorum*): space is a property attached to finite beings; in the case of infinite space God would have created infinite worlds. Similarly time is a consequence of moving beings: if time would have exist before creation, immutability of God would be in danger.

Leibniz proposed the relational theory of space and time based in theological considerations: space and time can not be infinite substance, because the world would be equal or identified with God; and also space and time could not be an attribution of created beings, because, if these beings are moving, in the motion the old position is lost. Therefore the only possibility is to accept the theory that the space and time is nothing more that a set of relations among things.

Leibniz was inspired in the Islamic doctrine of Kalam about the atomistic theory of space and time, as can be proved from the remarks written in the book of Maimónides «Guide for the Perplexed», although this doctrine was not considered of theological grounds [58].

#### 14.2. *Relational vs. absolutist theories*

The principal theories about the structure of space and time are not contradictory between themselves, but can be accepted from different frames of reference. The relational theory can be ascribed to space and time of created world, and the absolutist theory refers itself to the immensity of God who is omnipresent to all his creatures. From this last presupposition one can not identify the infinity of geometrical space with the immensity of God. In fact, the geometry can describe unlimited space with increasing capacity of new spaces, but never with an infinite actuality. The unlimited potentiality of geometrical spaces is only a broken image of the infinity of God in human spirit. Through His infinity God is non only present to all the creatures in order to constitute with His omnipresence the space of the creation, but also is condition of possibility of all the human conception about the special circumstances by which things are separated and connected among themselves.

### 14.3. *Simultaneity and eternity*

The problem of simultaneity in modern physics is a consequence of the synthesis of space and time advocated by Minkowski in 1908, a century ago, as a unified way of considering the Lorentz transformation of space-time coordinates. But according to the theory of relativity the simultaneity is impossible, generally speaking, between two observers in different inertial systems. By the contrary there can be two non simultaneous events in one inertial system that can be observed simultaneous in other inertial system. In the case of human conscience of time we can perceive as simultaneous different experiments in the past, in the present and in the future.

The presence of created beings in front of God must be considered also as transtemporal, that is to say: created beings belong to different times, but for God every creature remains always present to God. The eternity of God doesn't need any kind of memory or expectation, because His whole nature is simultaneous to all the events in the Universe. The eternity is the presence of life in its totality, not in the sense of a presence that is partially limited to the past and to the future but a transtemporal presence that doesn't have a future different from itself. The eternity is not a concentrate of time; the other way round, time with its three components —future, present and past— proceeds from the eternity.

### 14.4. *The future and the action of the Spirit of God*

According to S. Agustin's doctrine God had produced the time together with the creatures. Time can not be derived from eternity, although this is necessary for the coherence of different parts of time, in particular, past, present and future. The world was beloved by God, therefore space and time was also beloved by God. Can it be deduced that the finitude of creatures is always connected with time?

There are two answers: time is connected to creatures because presence is rooted into the past; by the other side, christian expectation is oriented to an end of time when the resurrection of the dead will take place.

The future is very important to the created beings from two reasons: firstly, their conservation, formation and consummation are depending from the future, that they do not control completely; secondly, their autonomy or independence from their divine creation, by which the creature is compelled to autodissolution. More important are the positive aspects of the future. This is «the field of the possible. It is thus the basis of the openness of creation to a higher consummation and the source what is new, i.e., of contingency in each new event» [59].

The Spirit of God manifests Himself in the creative power of the future, as field of possibility. This immersion of eternity of God in time has to be understood from the theological content as well as scientific domain. We can follow H. P. Dürr in his scientific approach that is based in the probabilistic interpretation of quantum mechanics: «Dürr first related quantum indeterminacy

to the concept of possibility, and this again to the future aspects of events, so that the future as the realm of the possible stands in contrast to the past as the realm of the factual and the present as the point when possibility becomes factual... this description gives the impression of a movement that comes from the future to the present, runs its course in the present, and is then fixed in the past» [60].

These physical interpretations help us to understand the theological idea of a dynamics of the divine Spirit, who, as a force of the future is acting in all events of the present. But the creating dynamics of the Spirit of God is not only the origin of temporal events but also of permanent ones, because the future that is controlled by the dynamics of the divine Spirit introduces the eternity of God in time. The action of the Spirit of God is to give to the creatures duration by a share in eternity and to protect them against the tendency to desintegrate that follows from their independence.

## 15. TORRANCE AND THE ONTOLOGY OF SPACE-TIME

Now we present a second theologian, Thomas Torrance in his book «Space, time and Incarnation» [61], where he develops the ontology of space-time in connection with the transcendence of God. Because the transcendence can not be tested with any human experiments, some epistemological presuppositions should be accepted. The point of departure is the creation because space-time is originated with the creatures. The space-time is image of the rationality of God in His creative activity and also it is the vehicle through which God became man. Creation and Incarnation are the axis where God and man meet together. In this axis we can find two system of coordinates: two horizontal coordinates that correspond to the space-time and one vertical coordinate that represents the transcendence of God. The intersection of these two systems of coordinates depends on the particular frame: Jesus Christ (theological frame), observational and geometrical models (physical frame), complementary language (mathematical frame).

### 15.1. *Epistemological questions about our knowledge of God*

If we want to talk about the theological aspect of space and time we have to accept that God has introduced Himself in the life of man and has taken the properties of human nature, in particular, the space and time of men.

In order to overcome these difficulties Torrance has proposed three epistemological principles:

- i) Whenever we talk about God's life and God's creation we have to refer to Him who transcends all time and all ages, and we must be careful of those expressions about the Son of God, such as «there never was a time when He did not exist» because they apply finite concepts to God.

- ii) There is an abyss between our knowledge and the reality of God such that Karl Barth claims: «How do we come to think, by means of our thinking, that which we can not think at all by these means». And so Barth insists that in the knowledge of God we can not raise questions about His reality from some position out side of Him, in other words, the true knowledge of God must come from the reality and the grace of the known object.
- iii) Torrance claims it is still possible to ask questions about the nature of God only within the circle of the knowing relationship in order to test the nature and the possibility of the rational structures within it. «Here we seek to bring to light the rational grounds upon which our knowledge claims to rest, either to establish it evidentially upon those grounds in such a way as to exhibit a thorough going consistency between our understanding and that into which we inquire, or to use the rationality that comes to light and the coherence of our operational structure to enable us to discriminate between reality and fictions» [62].

### 15.2. *Creation help us to understand rationality*

The Christian theology tells us that God made freely the Universe out of nothing with an immanent rationality making it knowable. The world is to be understood as subsisting in His creative Word. When we look to the Universe we can understand it because it has an inherent intelligibility owing to the relation of creative freedom between God and the creation. This rationality is grounded only in God alone, and for this reason the man can not understand God and His creation completely. «It is for this very reason, namely that the creation acquires its rationality in God's creative comprehension of it, that it is constituted and enabled to be the rational medium through which God speaks to us and makes Himself known, and in which once and for all His own eternal Word has become man» [63].

The space and time play an important role in the intelligibility of the created world, because they are created with the world and without them the world becomes disconnected from the transcendence of God. In fact, as Torrance says, space and time in the creation act as orderly function of contingent effects. In other place later he conceives the space and time as «a continuum of relations given in and with created existence and as the bearers of its immanent order» [64]. If there were no space and time the universe wouldn't be determined and intelligible and capax of formalization.

When God decides to become man and therefore, to live in the creature world he was freely submitted to the spatial and temporal conditions and for this reason he was inside of the space-time structures of the Universe. How can be considered the relations between the rationality of God independent of space and time and the rationality of the created world immersed in these spatial and temporal structures? This is equivalent to the question: what is the relation between the Incarnation and the nature of space and time?

### 15.3. *Incarnation and the nature of space-time*

Torrance presents four answers:

- i) «*Finitum capax infiniti*». In the Incarnation the Son of God enters into the realm of space and time which is finite and limited because it is created. But according to S. Thomas the Son of God became man without leaving the control of the Universe, therefore enlarging the limits of the receptacle of space and time, making the door open for the heresy of monophysitism.
- ii) «*Infinitum capax finiti*». This interpretation is based in the duality theory that the world consists in two parts: the container and the things that are contained. The container corresponds to Newtonian absolute space. The Son of God assumed the container and made it infinite in order to embrace all the existing beings in the Universe. But this hypothesis leads to contradiction because it is impossible to be an universal receptacle and at the same one of the particular beings within this receptacle.
- iii) God has endowed the creation with His own rationality and so space and time have relation with the activity of God. Space and time is the rational medium through which God is revealed and therefore all theological statements must find its intelligibility in the Being of God. According to Origen's mind this fact presupposes the coexistence of the Universe in the mind of God. In the line of Milne and Eddington the connection of the rationality of nature and the transcendence of God requires a necessary relation between them, from which the laws of nature could logically be derived.
- iv) The structures of space and time are created forms of rationality different from eternal rationality of God. These forms of rationality, are grounded in the divine Being but God remains free for creating them. The Incarnation of the Son of God in the created space and time assumes this reality but is different from it. This position of Anselm, Duns Scotus, Pascal and Karl Barth open the way to the idea that God's mind remains inscrutable. The answer to this difficult comes from the relation of freedom and necessity between God and the rationality of created beings. The world doesn't have the necessity God has in Himself. But once the world is created it has the contingent necessity, that can be compared with variational principle in physics by which once the minimal path is found all the rest are pure possibilities and the first one remains necessary but contingent.

From all these four possibilities of relation between Incarnation and space and time Torrance concludes: «In this way the Incarnation together with the creation form the great axis in God's relation with the world of space and time, apart from which our understanding of God and the world can only lose meaning» [65].

#### 15.4. *A definition of space-time*

Torrance speaks very frequently of a relational theory of space and time in the direction of Leibniz's philosophy as opposed to Newton's absolute idea of space and time (remember the quotation in Ref. [64]). After he has introduced the four types of connections between Incarnation and created rationality of space and time he gives a definition of them: «space and time must be conceived as the structural functioning of contingent event... space must be defined in terms of bodies or agents conceived as active principles, making room or creating space for themselves in the universe...» [66].

Later on Torrance completes his definition of space: «In physics, this means that geometry cannot be pursued as an axiomatic deductive science detached from actual knowledge of physical processes or be developed as an independent science antecedent to physics, but must be pursued in indissoluble unit with physics, as the science of its inner rational structure and as an essential part of empirical and theoretical interpretation of nature» [67].

From these finite structure of space and time Torrance develops the interaction of God with us through the medium he has chosen to communicate Himself to this world «The interaction of God with us in the space and time of this world sets up, as it were, a coordinate system between two horizontal dimension, space and time, and one vertical dimension, relation to God through His Spirit» [68].

Jesus Christ can not be considered developing a relational structure of space and time constructed by others, but organizing them around Himself, and «generating within these connections His own distinctive and 'continuous space-time track', and forming a moving and creative center for the confluence of world-lines within the plenum of space-time» [69].

Torrance has developed a relational concept of space and time in accordance with the nature and acts of man, and has described the two ways of understanding with two sets of coordinates, vertical and horizontals, the first one corresponding to the infinite transcendent knowledge of God, the second one corresponding to the human understanding that received his finite knowledge through the regularized structure of space and time.

#### 15.5. *Intersection of horizontal and vertical coordinates*

The two ways of knowing correspond to two different rationalities. One is transcendent and infinite, the other is contingent and finite but has its support in the first one because it is created by God.

Similar situation occurs in physics where the language derived from observations —take quantum mechanics— is different from the language used in hypothetical-deductive models —take general theory of relativity— although both refer to the same phenomena. Torrance has proposed three ways to explain the intersection of the vertical and horizontal coordinates that represent the infinite rationality of God and the created rationality of man through the structure of space and time:

- 1) The first answer is taken from the doctrine of the Church Fathers, according to whom Jesus Christ's space-time is the place where God is present in our world. «Jesus Christ is the place of contact and communication between God and man in a real movement within physical existence, involving interaction between God and nature, divine and human agency» [70].
- 2) Physicists use topological languages to represent the connection between the dynamical and geometrical aspects of things. Greek Fathers talk about mental place where energy —of no observable activity— links physical with divine space (what Whitehead call 'passage beyond nature').

«The topological language can be used to express the relation of place in the physical sense to the whole of space-time through the consideration of some field of energy or action... It attempts to rise above the level of thought in which we think... in simple geometry of pattern of corpuscular distribution to the level in which we think of distinguishable situations and positions of things» [71].

- 3) A third way to solve the problem of using different languages —observational and theological— is through the Gödel's theorem that is used in the case of two different languages. Gödel showed that in any consistent formal system there must be some propositions which can not be proved or disproved within the system. The incompleteness can be solved by amplification onto a different level of formal system that is coordinated with the first one. The application of this multiplicity of formal-logical systems to the understanding of space-time means that there are, at least, two levels of languages: one corresponding to ordinary language and other defined within the formal calculus, but «they must be coordinated through the hierarchical structure that connects the different levels» [72]. Similar consideration should be applied to the intersection of theological and ordinary languages about God.

## 16. CONCLUDING REMARKS

We have presented some relational theories on the structure of space-time. They start from some elementary objects (Penrose's spin units, Finkelstein's monads, Weizsäcker's urs, Sorkin's vertices, Markopoulou's events) that can interact among themselves (spin networks, checker board, processes, causal sets, spin foams) because they are endowed with causality (causal sets, quantum causal histories, causal spin foams) and the quantum effects are taking into account. All the elementary objects and their properties belong to the theoretical level but they can be interpreted in the ontological one).

We have presented also our model on the nature of the space-time that turns out to be relational, discrete, causal and quantum, the ontological background of which consists on the causal interactions of individual beings that we call «hylions» [73]. Our model can be considered a particular example of causal spin foam models.

In the final section we have implemented the physical and philosophical models with theological considerations, taken from Pannenberg and Torrance. These theologians have stressed the finite nature of space-time that started with the creation, although it participates of the infinite rationality of God. The rationality of space-time is the place where the creatures communicate among themselves and with God and where the Incarnation of God takes place. Due to the infinite potentiality of created beings and the infinite freedom of God, the actual structure of space-time depends only on God's decision.

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